Seismic Source Theory: forward and inversion

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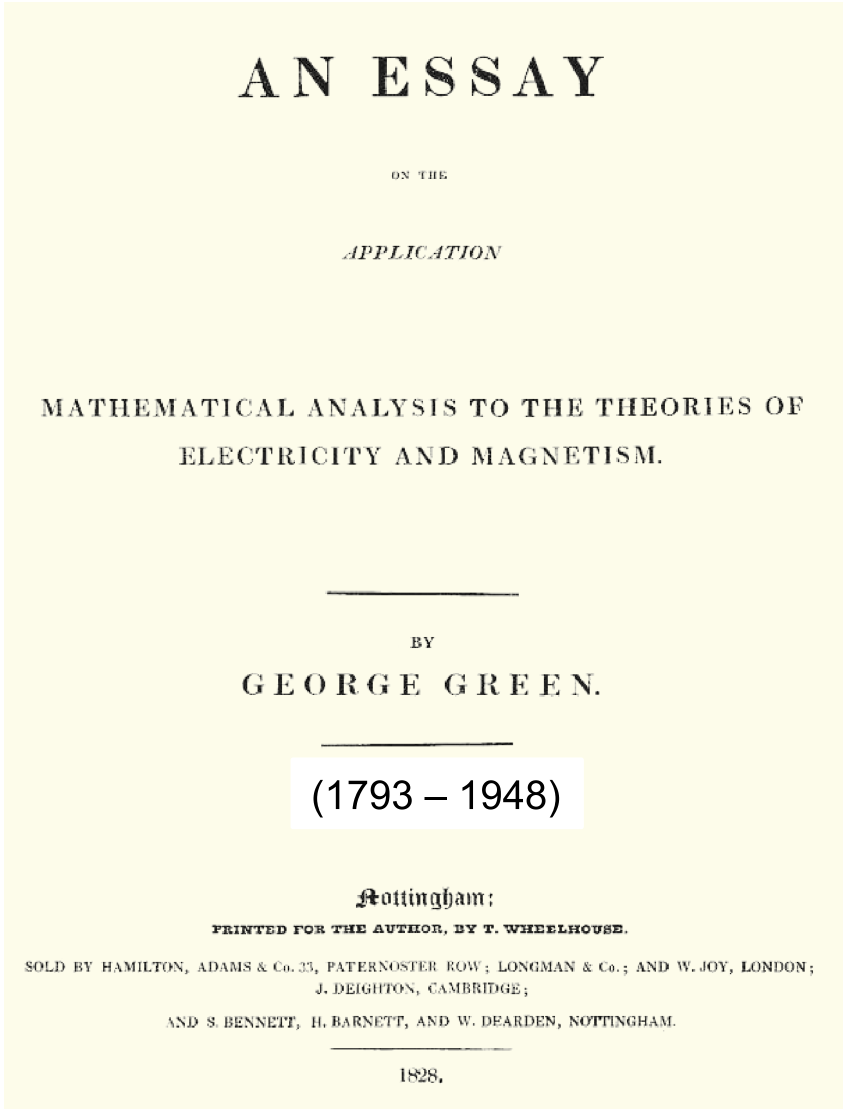
Content

1. Forward problem: basic equations and terms
   - Green functions
   - near and far field representations
   - moment tensor
2. Physics of rupture
   - variation of rupture and slip
   - approximate model (Eikonal source model)
3. Representation of extended sources
   - frequency and time domain directivity
   - how to resolve higher order terms

Green’s mill in Nottingham



AN ESSAY

ON THE

APPLICATION

OF

MATHEMATICAL ANALYSIS TO THE THEORIES OF
ELECTRICITY AND MAGNETISM.

BY

GEORGE GREEN.

(1793 – 1948)

Nottingham;

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1828.

Green's theorem

source terms

1. Potential fields ϕ and G fulfill Poisson's equation

$$\nabla^2\phi = f \;\; and \;\; \nabla^2 G = \delta(r).$$

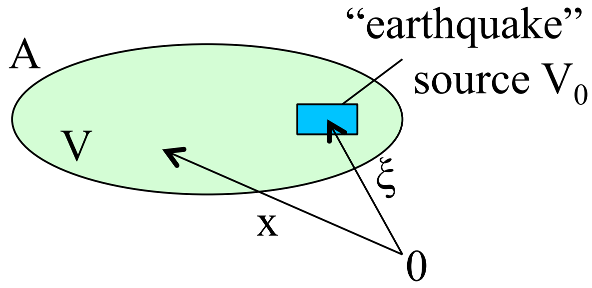
2. Apply chain rule for terms like

$$\begin{aligned}
\nabla \cdot (\phi \nabla G) &= \phi \nabla \cdot \nabla G + (\nabla \phi)(\nabla G) \\
\nabla \cdot (G \nabla \phi) &= G \nabla \cdot \nabla \phi + (\nabla \phi)(\nabla G)
\end{aligned}$$

3. Form difference

$$\int_V \phi \nabla \cdot \nabla G - G \nabla \cdot \nabla \phi \, dV = \int_V \nabla \cdot (\phi \nabla G) - \nabla \cdot (G \nabla \phi) \, dV = \int_A (\phi \nabla G - G \nabla \phi) \, \hat{\mathbf{n}} dA$$

specified source geometry



δ(x) ⇕   f(ξ) ⇕

$$\int_V \left(\phi\nabla^2 G \;-\; G\nabla^2\phi\right) dV \;=\; \int_A \left(\phi\nabla G \;-\; G\nabla\phi\right) \cdot\, d\mathbf{A}\,,$$

gives representation of field variable ϕ at x as

$$\phi(\mathbf{x}) \;=\; \int_V f(\boldsymbol{\xi})G(\mathbf{x},\boldsymbol{\xi})\,dV \;+\; \int_A \left(\phi(\boldsymbol{\xi})\nabla|_{\xi}G(\mathbf{x},\boldsymbol{\xi}) \;-\; G(\mathbf{x},\boldsymbol{\xi})\nabla|_{\xi}\phi(\boldsymbol{\xi})\right) \cdot\, d\mathbf{A}(\boldsymbol{\xi})\,.$$

Application I (V infinite): ⟹ Green function representation

$$\phi(\mathbf{x}) \;=\; \int_V f(\boldsymbol{\xi})G(\mathbf{x},\boldsymbol{\xi})\,dV$$

Application II (f zero): ⟹ Boundary element representation

Betti's theorem: "Green theorem for time dependent elasticity"

$$u_i(\mathbf{x},t) \;=\; \int_V f_j(\boldsymbol{\xi},t) * G_i^j(\mathbf{x};\boldsymbol{\xi},t)\,dV$$

$$+ \int_A [t_j(\vec{\xi},t) * G_k^j(\vec{\xi},\vec{x},t) - u_j(\vec{\xi},t) * T_k^j(\vec{\xi},\vec{x},t)]dA(\xi)$$

Green tensor ***G***

displacement **u**

force ***f***

traction fields ***t*** and ***T***

<u>Application I</u> (displacement and traction vanish on A): force vector representation

$$u_i(\mathbf{x},t) \;=\; \int_V f_j(\boldsymbol{\xi}) \,*\, G_i^j(\mathbf{x};\boldsymbol{\xi})\,dV \;=\; \int_V \int_{-\infty}^{t} f_j(\boldsymbol{\xi},\tau)\,G_i^j(\mathbf{x},t;\boldsymbol{\xi},\tau)\,d\tau\,dV\;.$$

Example: Static point force representation (Somigliana solution)

$$u_i(\mathbf{x}) \;=\; F_j(\boldsymbol{\xi}_0)\, G_i^j(\mathbf{x}, \boldsymbol{\xi}_0)$$

displacement ***u***

force ***f***

Greens „function“ (GF) ***G***

But: internal sources (earthquakes) cannot be represented by single forces!

Internal point sources: GF expansion and moment tensor

Taylor's series expansion around centroid ξ_0 :

$$G_i^j(\boldsymbol{x};\boldsymbol{\xi}) \approx G_i^j(\boldsymbol{x};\boldsymbol{\xi_0}) + \frac{\partial}{\partial \xi_k} G_i^j(\boldsymbol{x};\boldsymbol{\xi})|_{\xi_0}\, \delta\xi_k$$

where $d\xi_k = (\boldsymbol{\xi} - \boldsymbol{\xi_0})_k.$

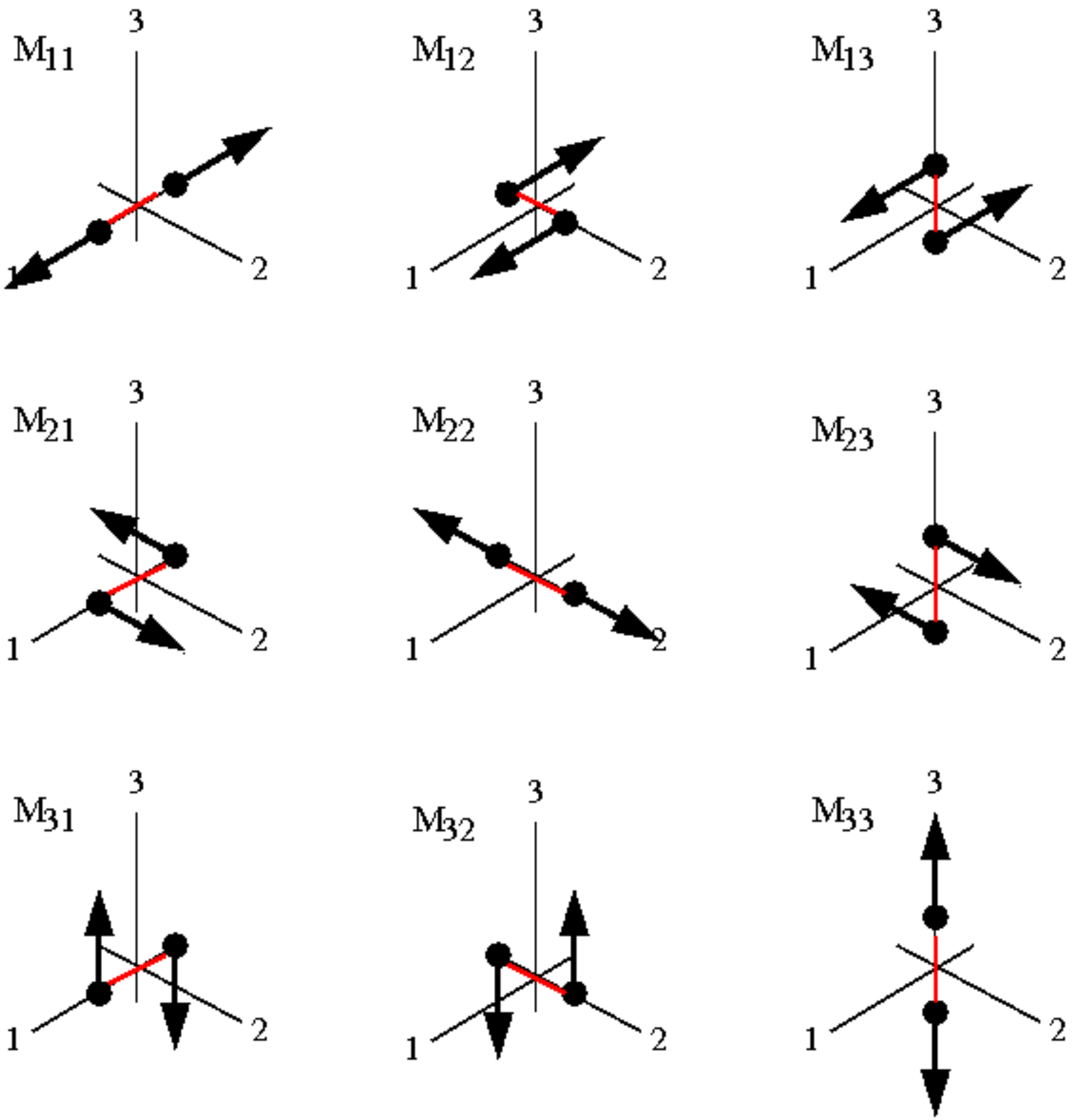
gives

„internal source" at centroid!

$$\begin{aligned} u_i(\mathbf{x}) &= G_i^j * \int_V f_j\, dV + G_{i,k}^j * \int_V f_j \delta\xi_k\, dV + G_{i,kl}^j * \int_V f_j \delta\xi_k \delta\xi_l\, dV + \dots \\ &= G_i^j * M_j + G_{i,k}^j * M_{jk} + G_{i,kl}^j * M_{jkl} + \dots , \end{aligned}$$

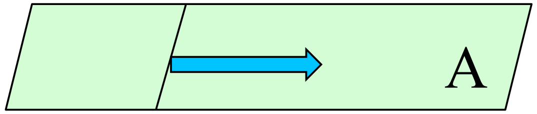
with generalized moment $$M_{jk_1 \dots k_l} = \int_V (\xi_{k_1} - \xi_{0k_1})(\xi_{k_2} - \xi_{0k_2}) \dots (\xi_{k_l} - \xi_{0k_l}) f_j(\xi) dV$$

Moment tensor; generalized force couples



- 3x3 matrix
- symmetric
- related to earthquake rupture

Moment tensor density *m* of dislocation source



$$u_n(\mathbf{x},t) \;=\; \int_A m_{pq}(\boldsymbol{\xi},t) \star G^p_{n,q}(\mathbf{x},\boldsymbol{\xi},t)\, dA$$

rupture plane

$$m_{pq}(\boldsymbol{\xi},t) \;=\; \mathcal{N}\left(\hat{n}_p \Delta u_q(\boldsymbol{\xi},t) + \hat{n}_q \Delta u_p(\boldsymbol{\xi},t)\right)$$

shear modul

slip vector

fault plane normal

Full space Green function

direction cosines

geometrical attenuation

radiation pattern term

$$
\begin{aligned}
M_{jk} * G^{j}_{i,k} \;=\; & \left(\frac{15\gamma_i\gamma_j\gamma_k - 3\gamma_i\delta_{jk} - 3\gamma_j\delta_{ik} - 3\gamma_k\delta_{ij}}{4\pi\rho} \frac{1}{r^4} \int_{\frac{r}{v_p}}^{\frac{r}{v_s}} \tau M_{jk}(t-\tau)\, d\tau \right) \\
& + \left(\frac{6\gamma_i\gamma_j\gamma_k - \gamma_i\delta_{jk} - \gamma_j\delta_{ik} - \gamma_k\delta_{ij}}{4\pi\rho v_p^2} \frac{1}{r^2} M_{jk}(t - \frac{r}{v_p}) \right) \\
& - \left(\frac{6\gamma_i\gamma_j\gamma_k - \gamma_i\delta_{jk} - \gamma_j\delta_{ik} - 2\gamma_k\delta_{ij}}{4\pi\rho v_s^2} \frac{1}{r^2} M_{jk}(t - \frac{r}{v_s}) \right) \\
& + \frac{\gamma_i\gamma_j\gamma_k}{4\pi\rho v_p^3} \frac{1}{r} \dot{M}_{jk}(t - \frac{r}{v_p}) \;-\; \frac{\gamma_i\gamma_j - \delta_{ij}}{4\pi\rho v_s^3} \gamma_k \frac{1}{r} \dot{M}_{jk}(t - \frac{r}{v_s}) \quad ,
\end{aligned}
$$

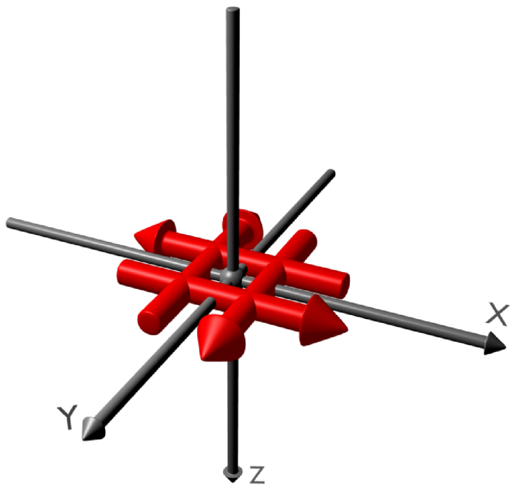
near field term (second and third lines)

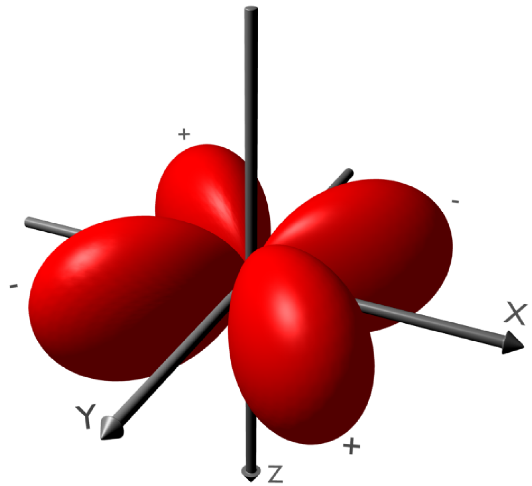
far field terms (last line)

in far field moment rate function

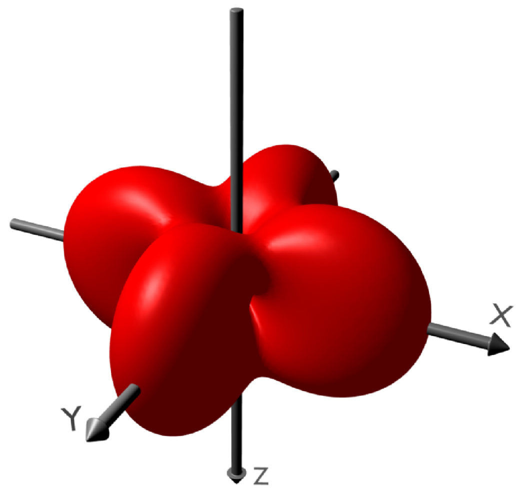
retarded time of P and S waves

Radiation pattern of Double Couple Source





P wave



S wave

Far field simplifications

$$u_i(\mathbf{x},t) \quad = \quad M_{jk}\, S(t) * \frac{\partial}{\partial \xi_k} G_i^j(\vec{x},t;\vec{\xi})|_{\vec{\xi}_0} \quad .$$

source time function

with $M_{jk}(t) = M_{jk}\, S(t)$

$$\frac{\partial}{\partial \xi_k} G_i^j(\mathbf{x},t') \quad = \quad \frac{\partial G_i^j}{\partial x_l}\frac{\partial x_l}{\partial \xi_k} + \frac{\partial G_i^j}{\partial t}\frac{\partial t'}{\partial \xi_k}$$

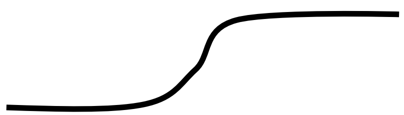
near field term

far field term

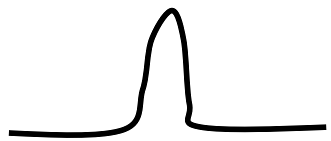
Far field representation

$$u_i(\mathbf{x},t) \;\approx\; M_{jk}\, S(t) * \dot{G}_i^j(\mathbf{x},t)\,\frac{\partial t'}{\partial \xi_k} \;=\; M_{jk}\, \dot{S}(t) * G_i^j(\mathbf{x},t)\,\frac{\partial t'}{\partial \xi_k}$$

near field: S(t)



far field: dS/dt (moment rate function)



Idealized moment tensor (MT) inversion: Note!

Fact 1:

The spatial point source MT-inversion is unique

Fact 2:

The spatially extended source problem is nonunique

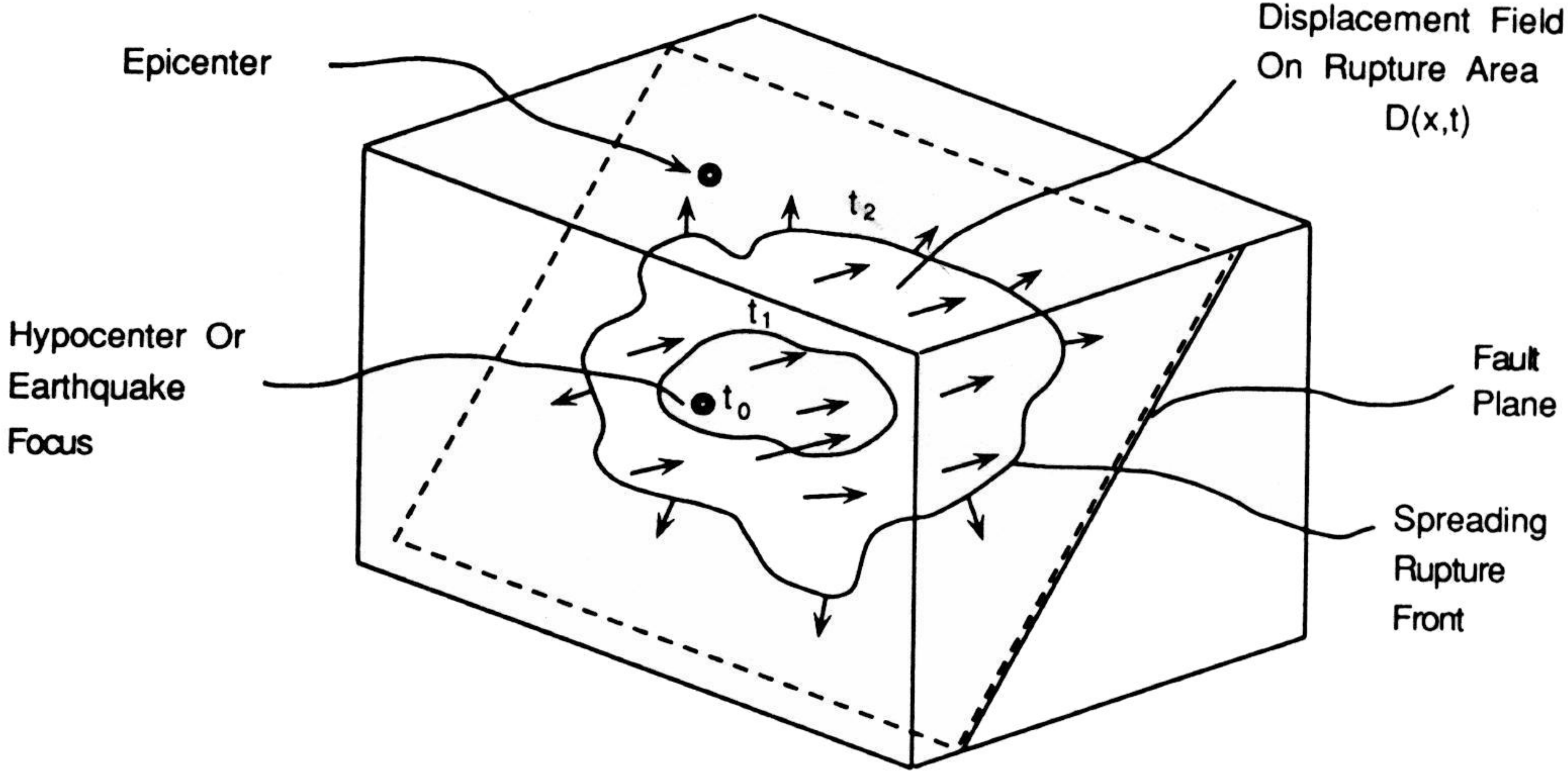
but:

The non-uniqueness can be solved if time-dependency of rupture is known/given (e.g. Bleistein and Cohen (1977). J. Math. Phys. 2, 5-26).

Memo plate (theory section)

- Green functions represent wave and displacement response to point loading of the Earth
- Moment tensor defined as generalized force couples
- Moment tensor representation can be given for:
    - point sources or extended sources
    - near field (may be time-independent) and/or far field

Earthquake rupture: phenomenology

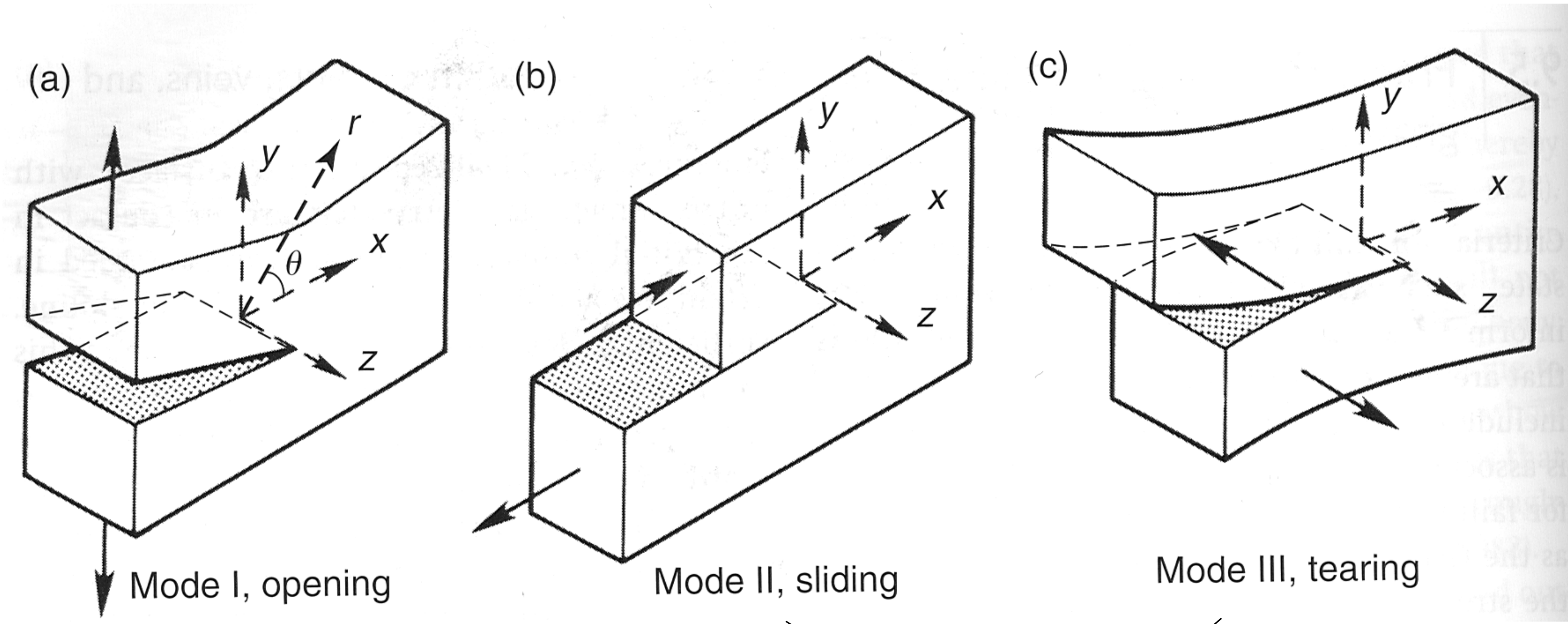


from Lay and Wallace (1995)

physical description of rupture

- rupture front (velocity and shape)
- rise time and slip function (temporal)
- healing front (velocity and shape)
- slip direction and slip pattern (spatial)
- rupture mode

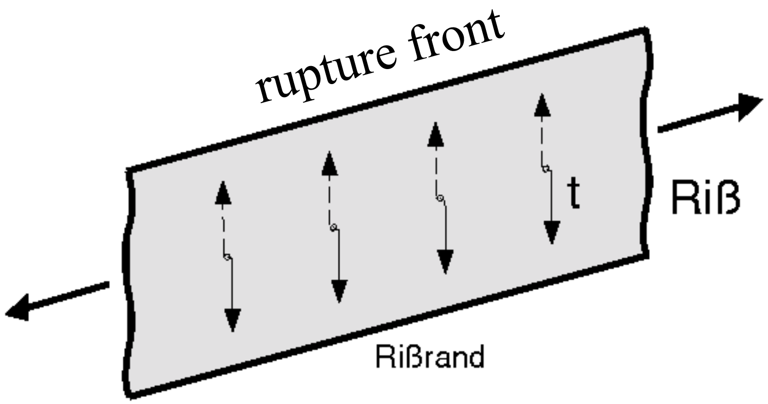
rupture modes



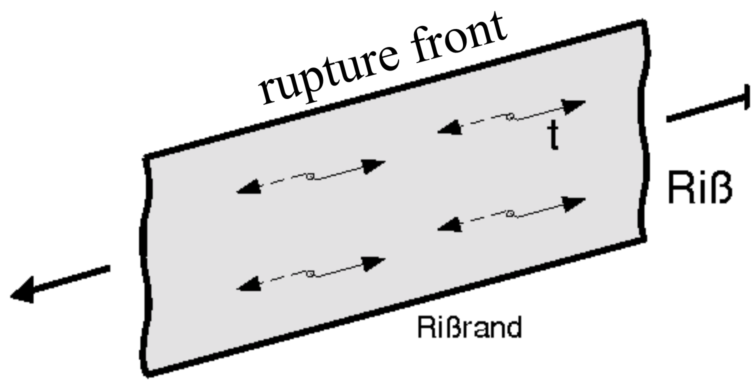
no friction

friction may be important!

1. “slip” of shear cracks without friction (3D simulations)



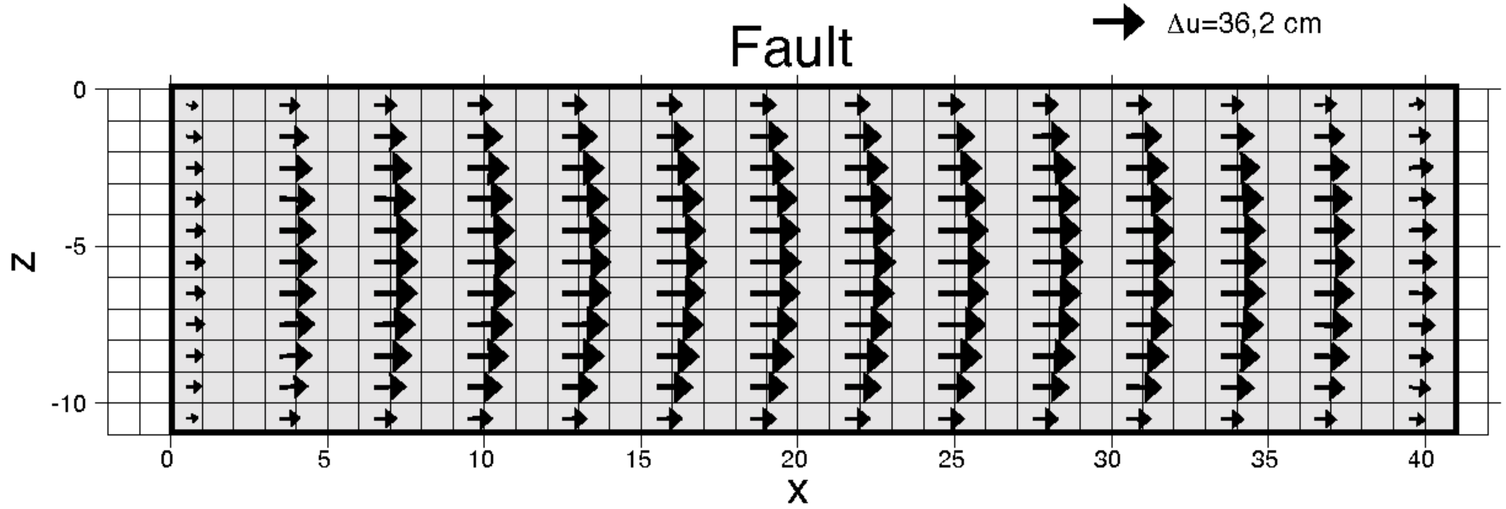
a. Mode II



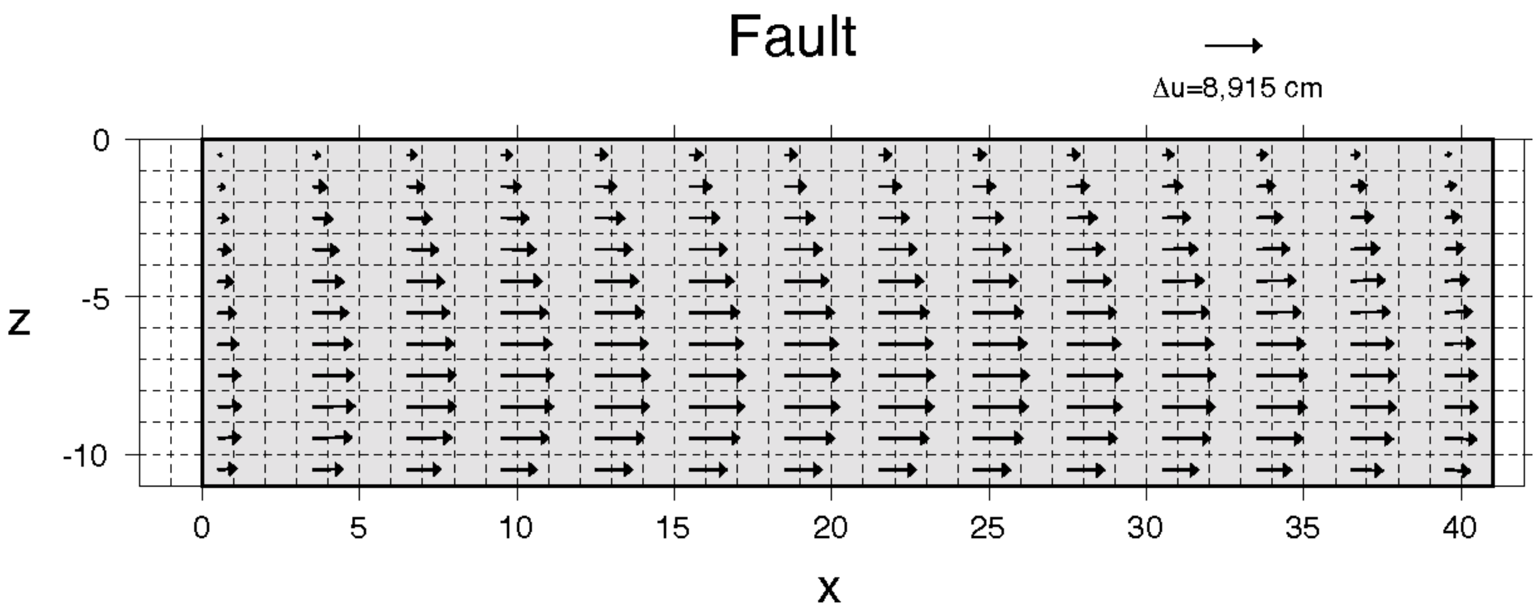
b. Mode III

shear stress drop is imposed !

Crack solutions of strike slip fault, <u>full space</u>

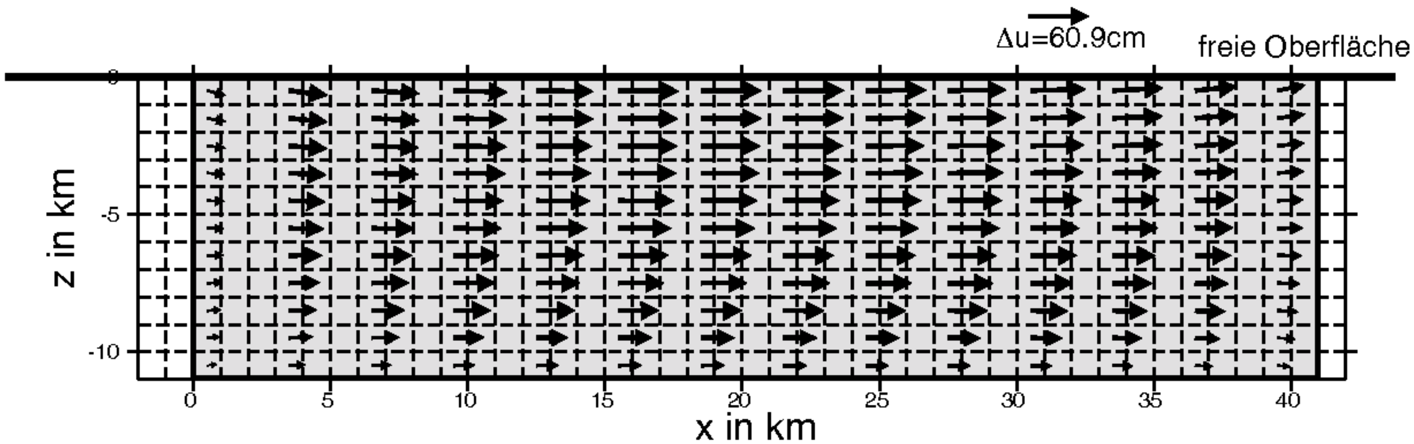


constant shear

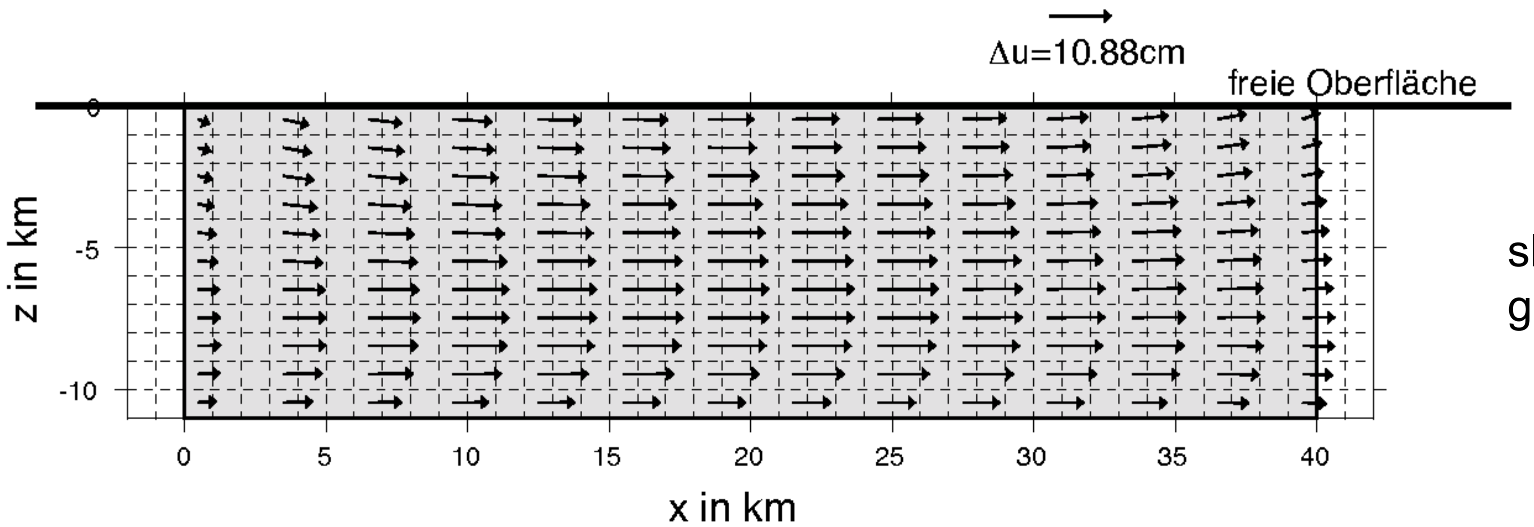


shear stress linearly growing with depth

Crack solutions of strike slip fault, half space



constant shear



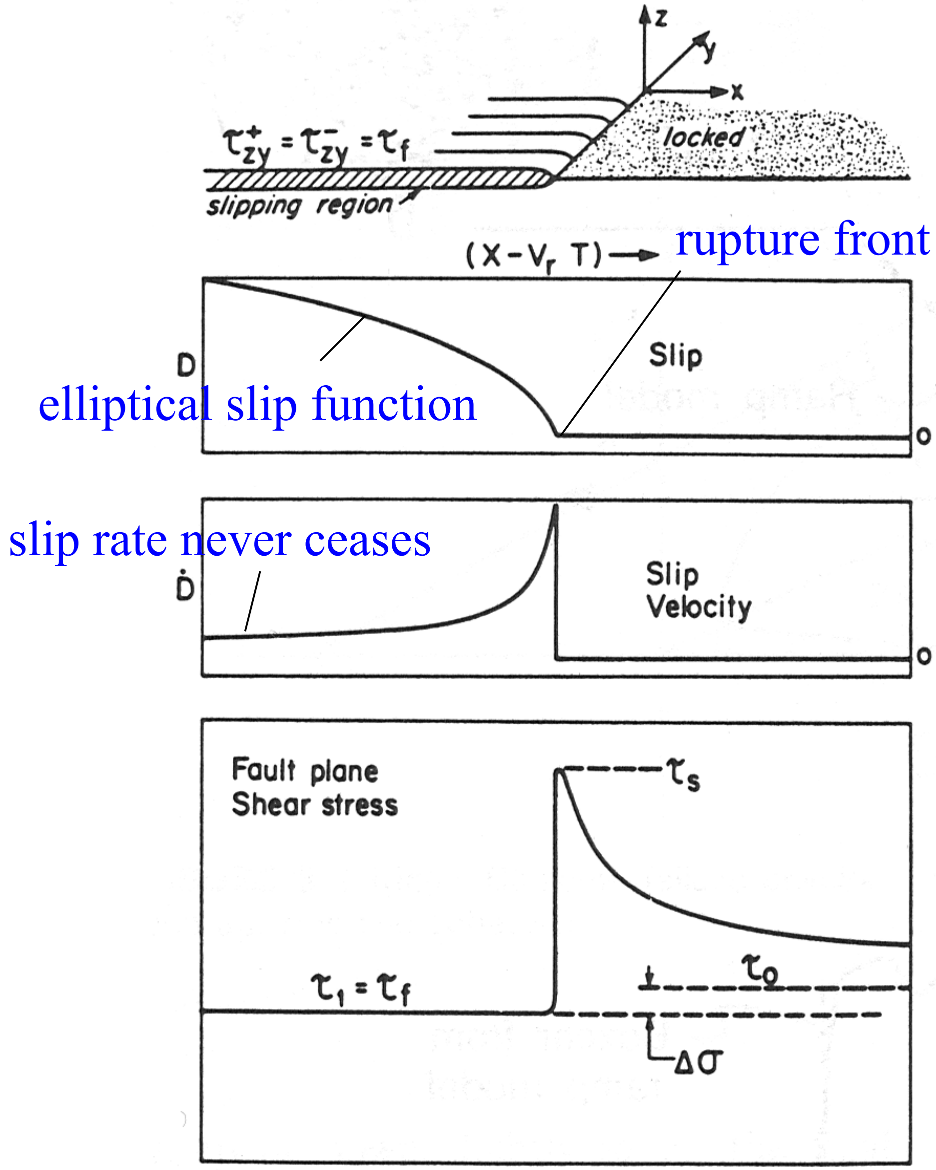
shear stress linearly growing with depth

Note: earthquake slip may vary at barriers and asperities

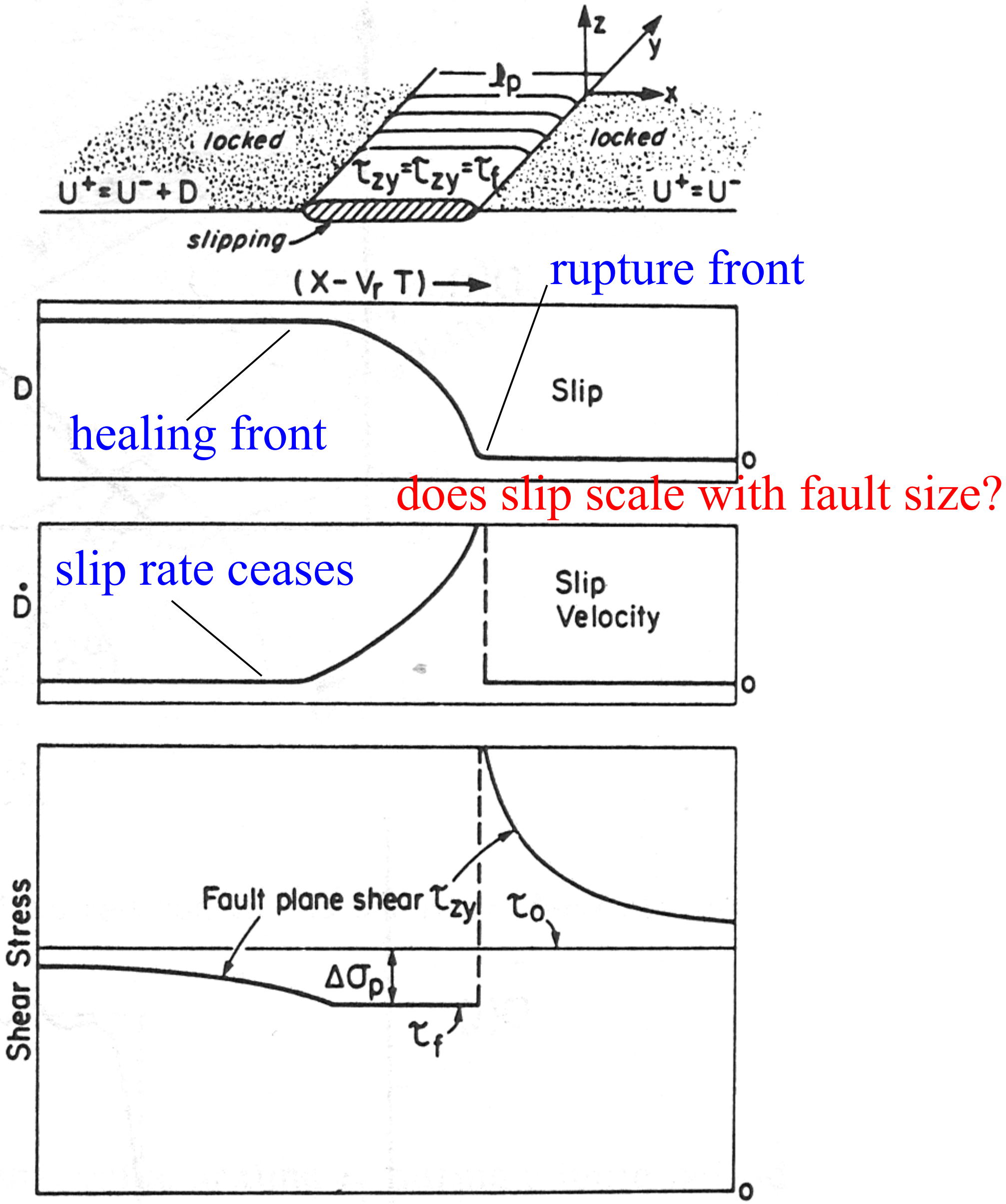
Barriers: patches of high strength (compare inclusion).

Asperities: patches of high stess. Larger shear stress before the earthquake. Points, where new rupture may nucleate (compare capillare)

2. slip models considering friction

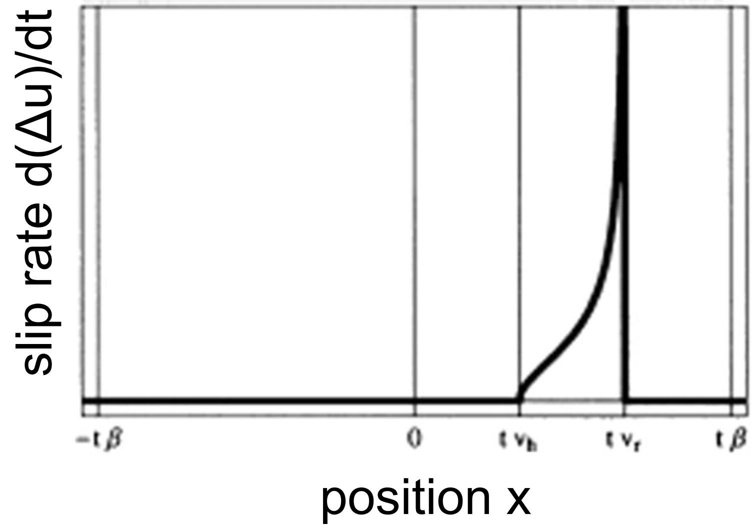


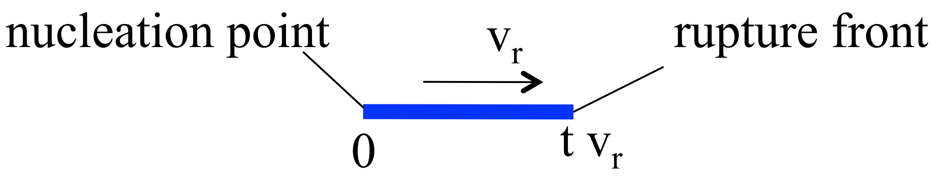
Kostrov (1966)



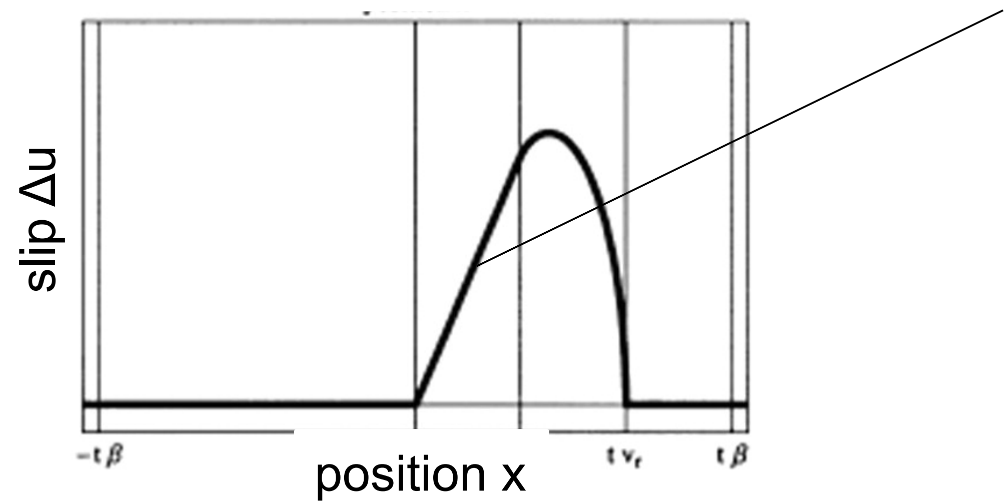
Self-healing slipping model, Heaton (1990)

Slip pulse model: slip rate and slip as a function of position x



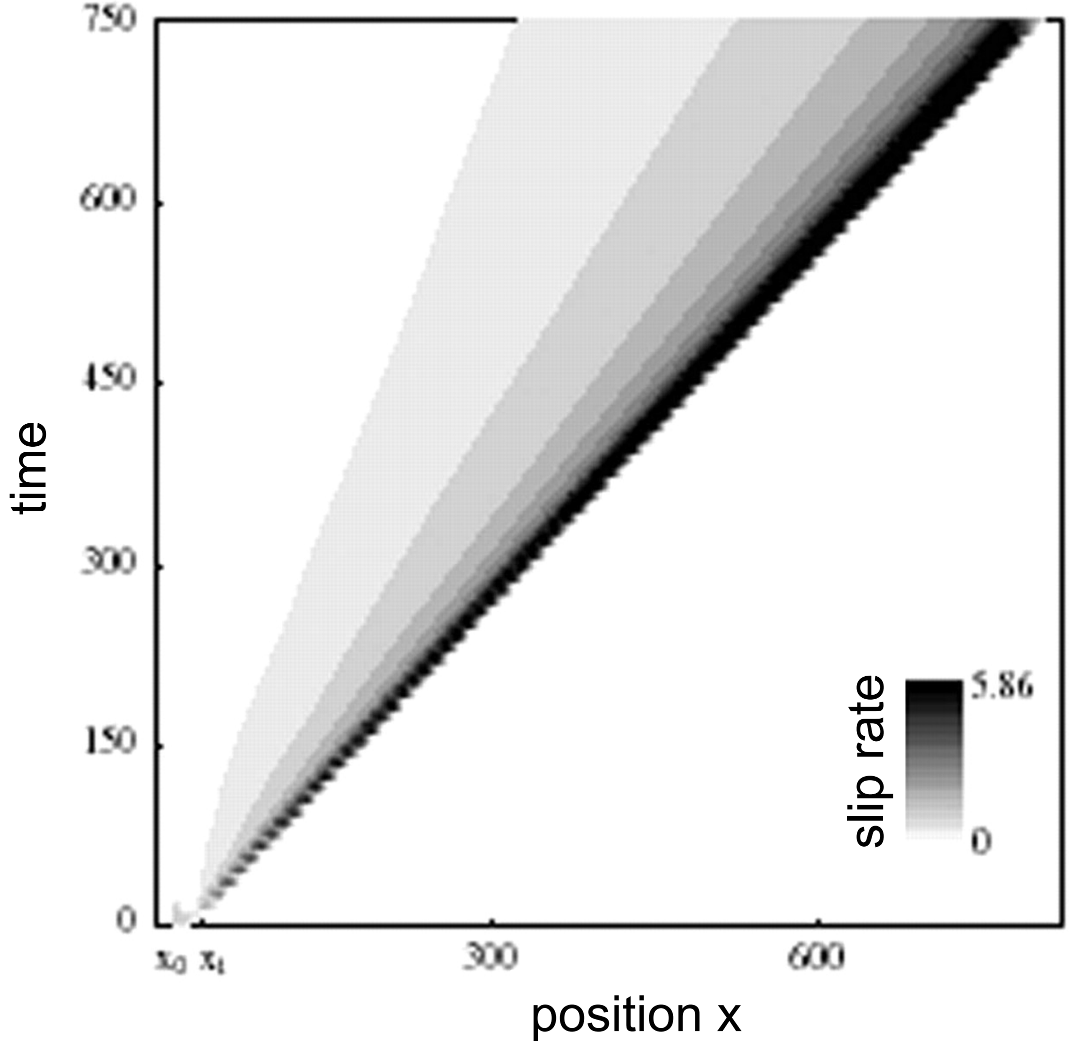


slip increases linearly from nucleation point to healing front !



Nielson and Madariaga (2003)

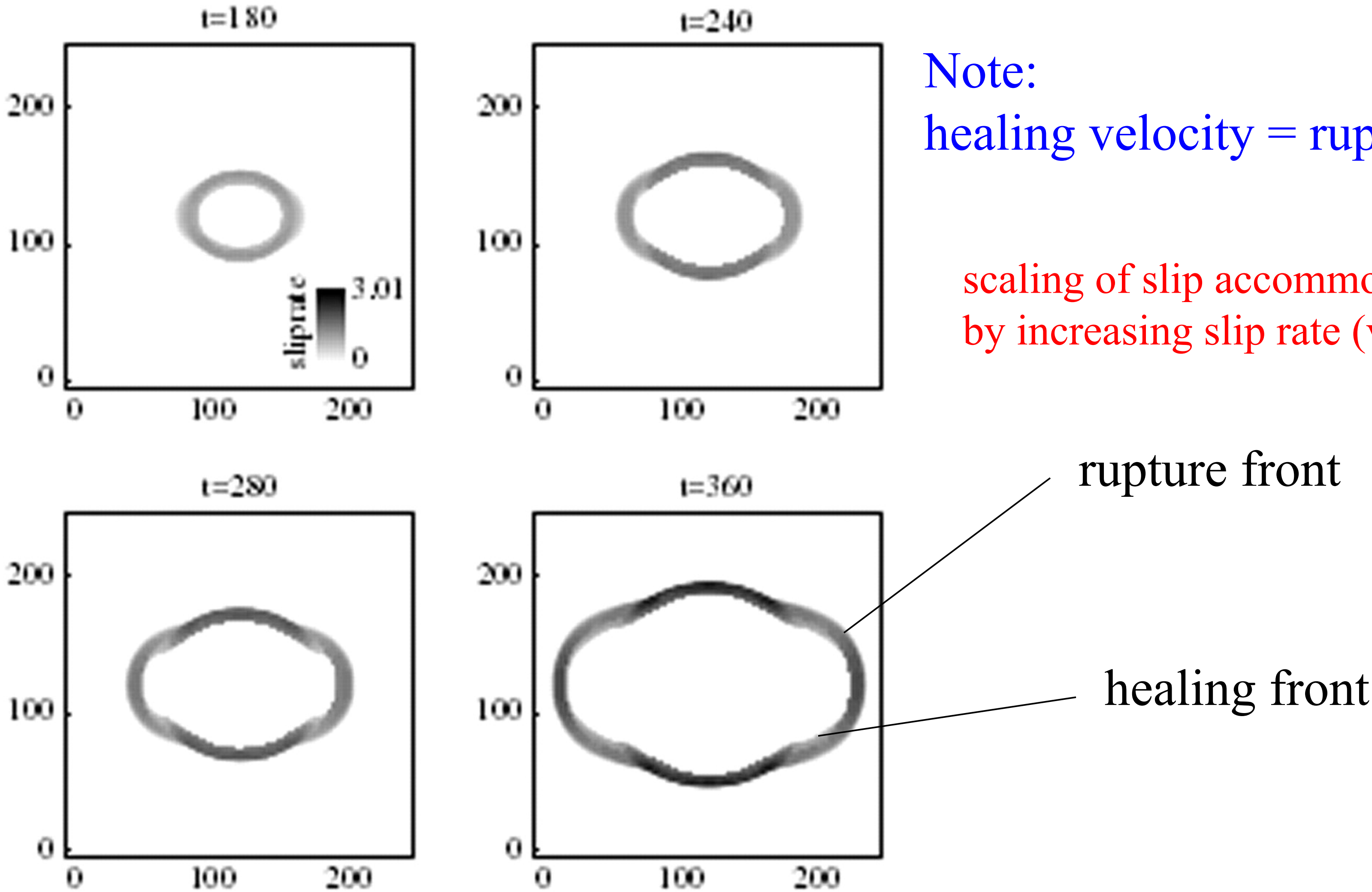
2D numerical models:
Slip rate as a function of time and position



scaling of slip is accommodated by increasing rise time ($v_r > v_h$)

Nielson and Madariaga (2003)

3D numerical modeling with weakening

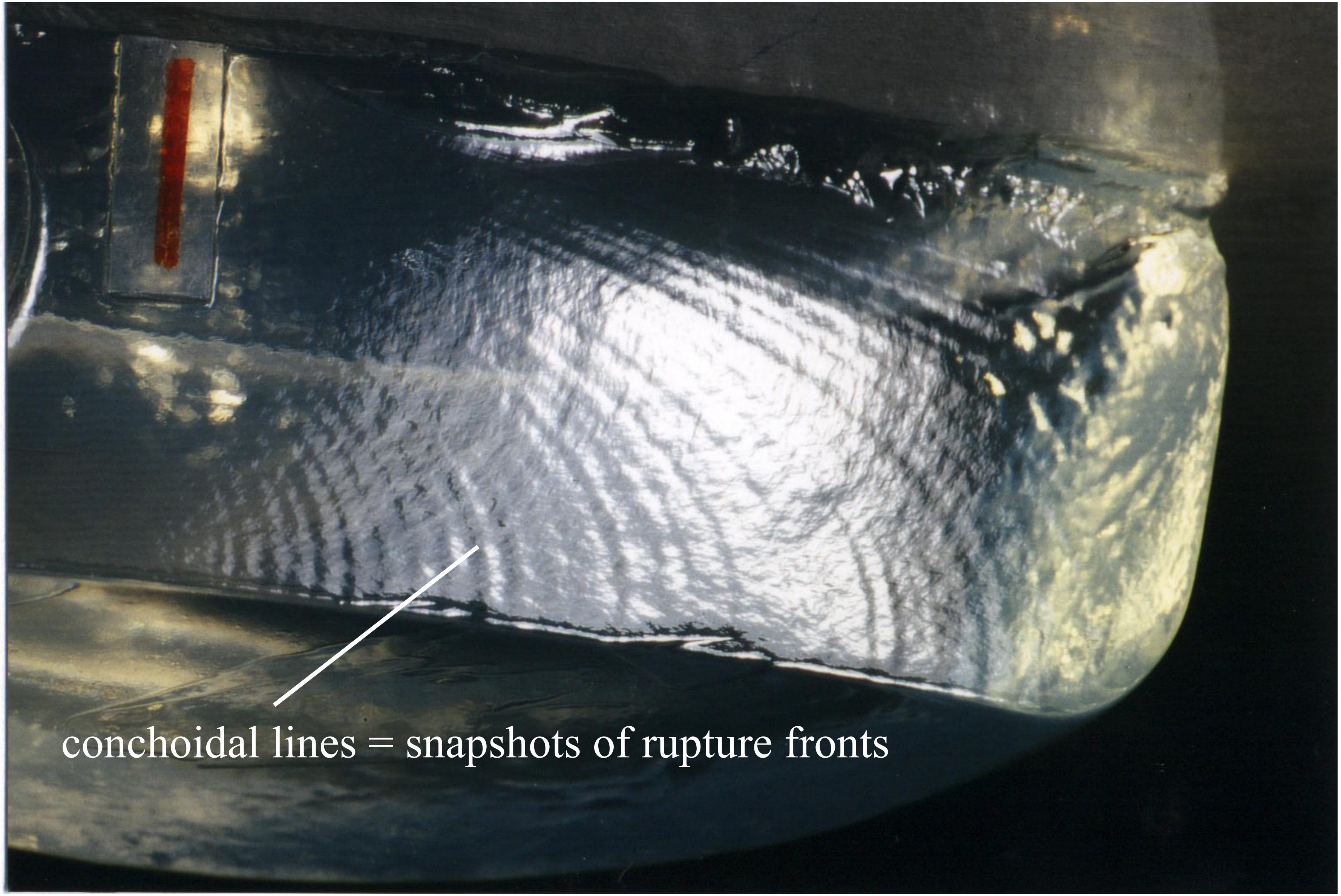


Note:
healing velocity = rupture velocity

scaling of slip accommodated by increasing slip rate ($v_r = v_h$)

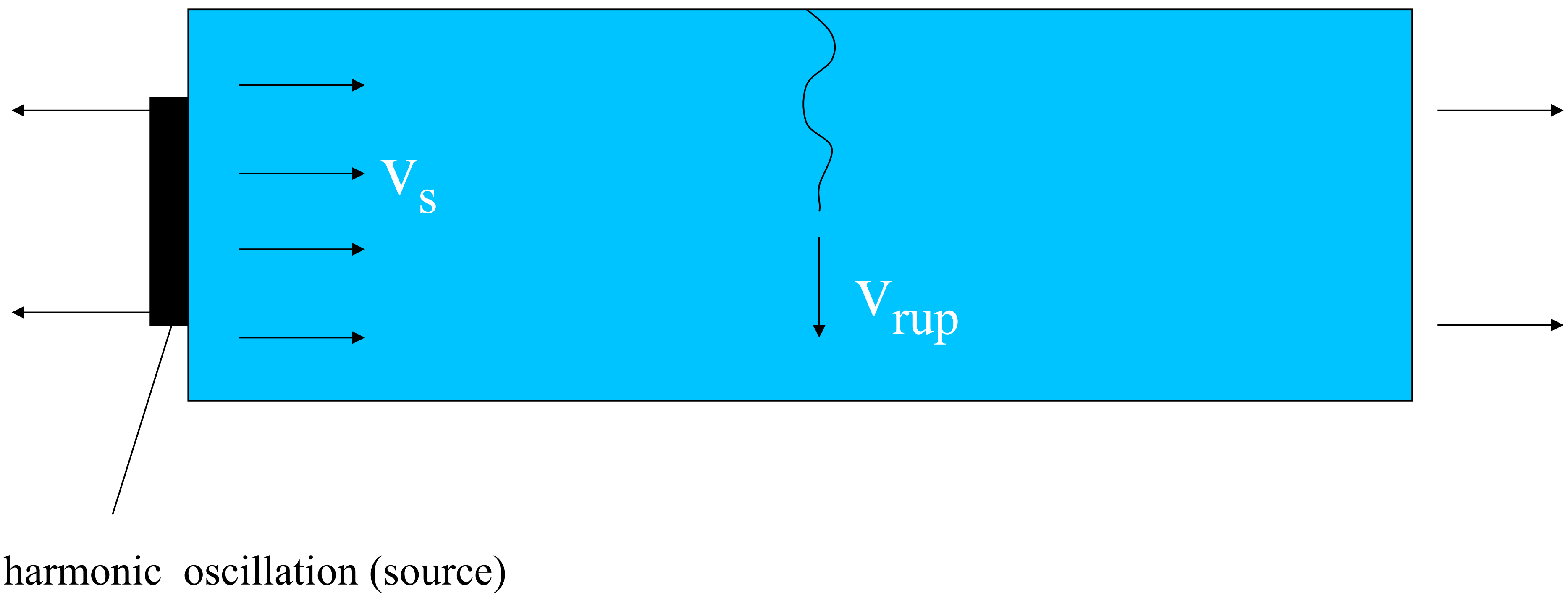
Nielson and Madariaga (2003)

3. how large is rupture front velocity

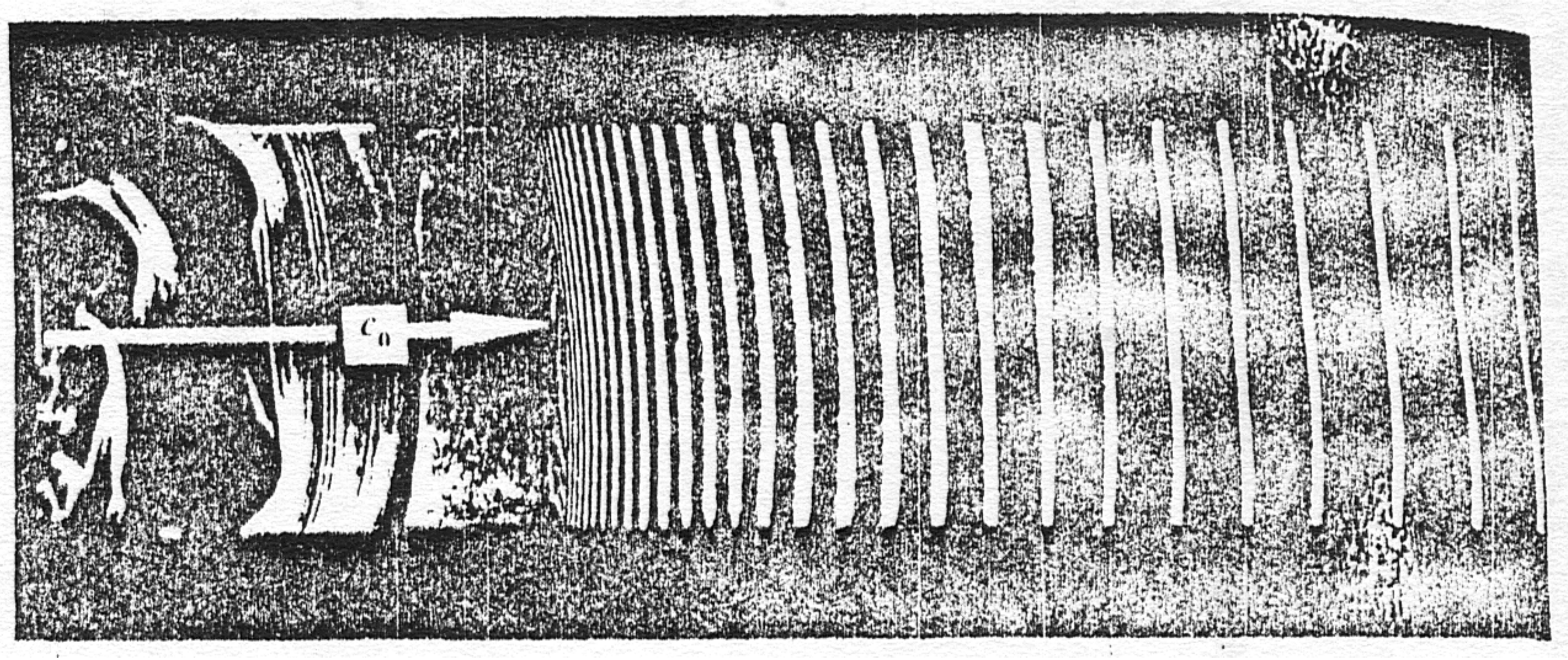


„shaking and quaking experiment by Gerhard Müller: tensile crack rupture in gelatine

rupture front modulation: sketch



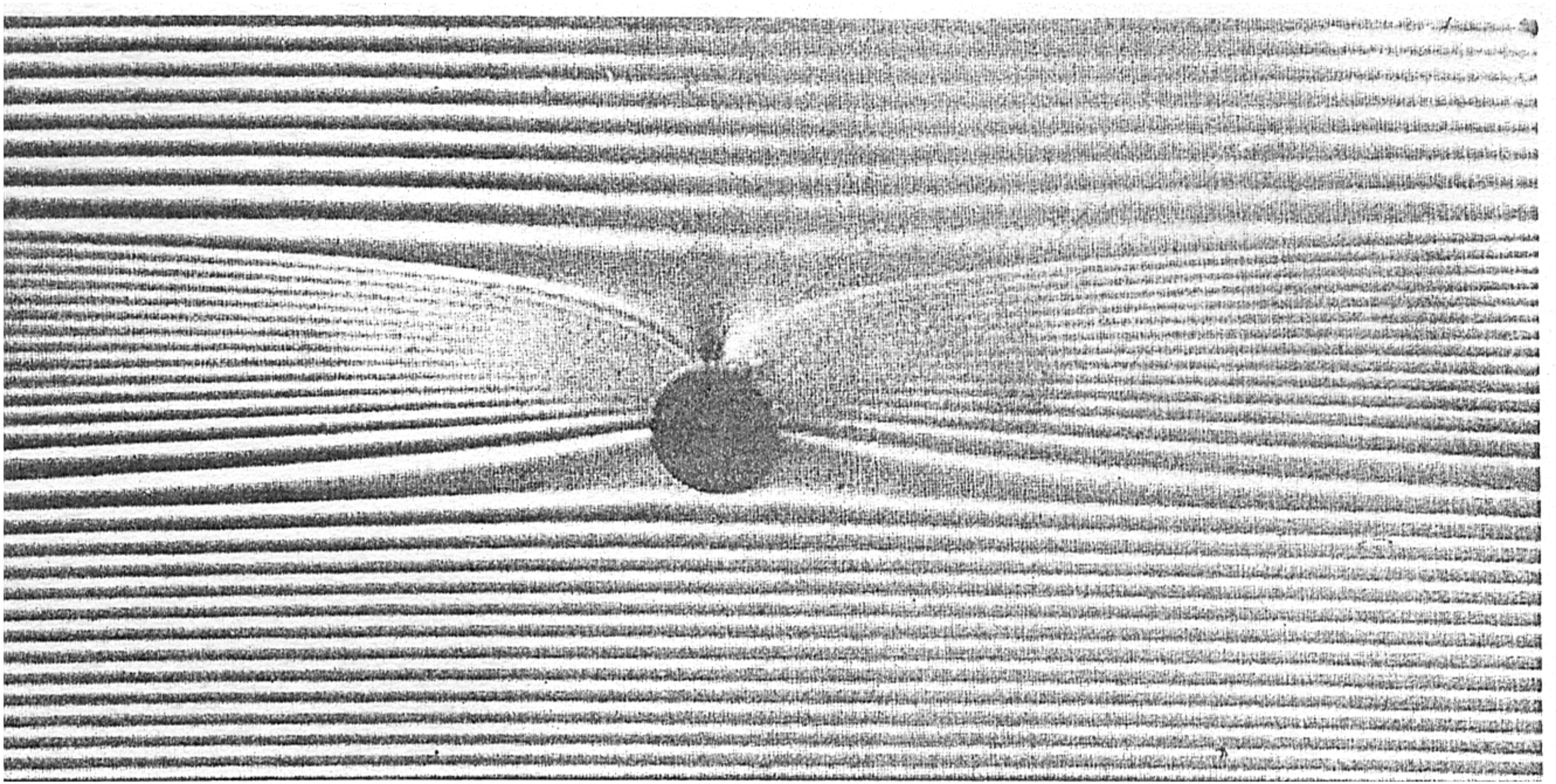
Lab experiments results



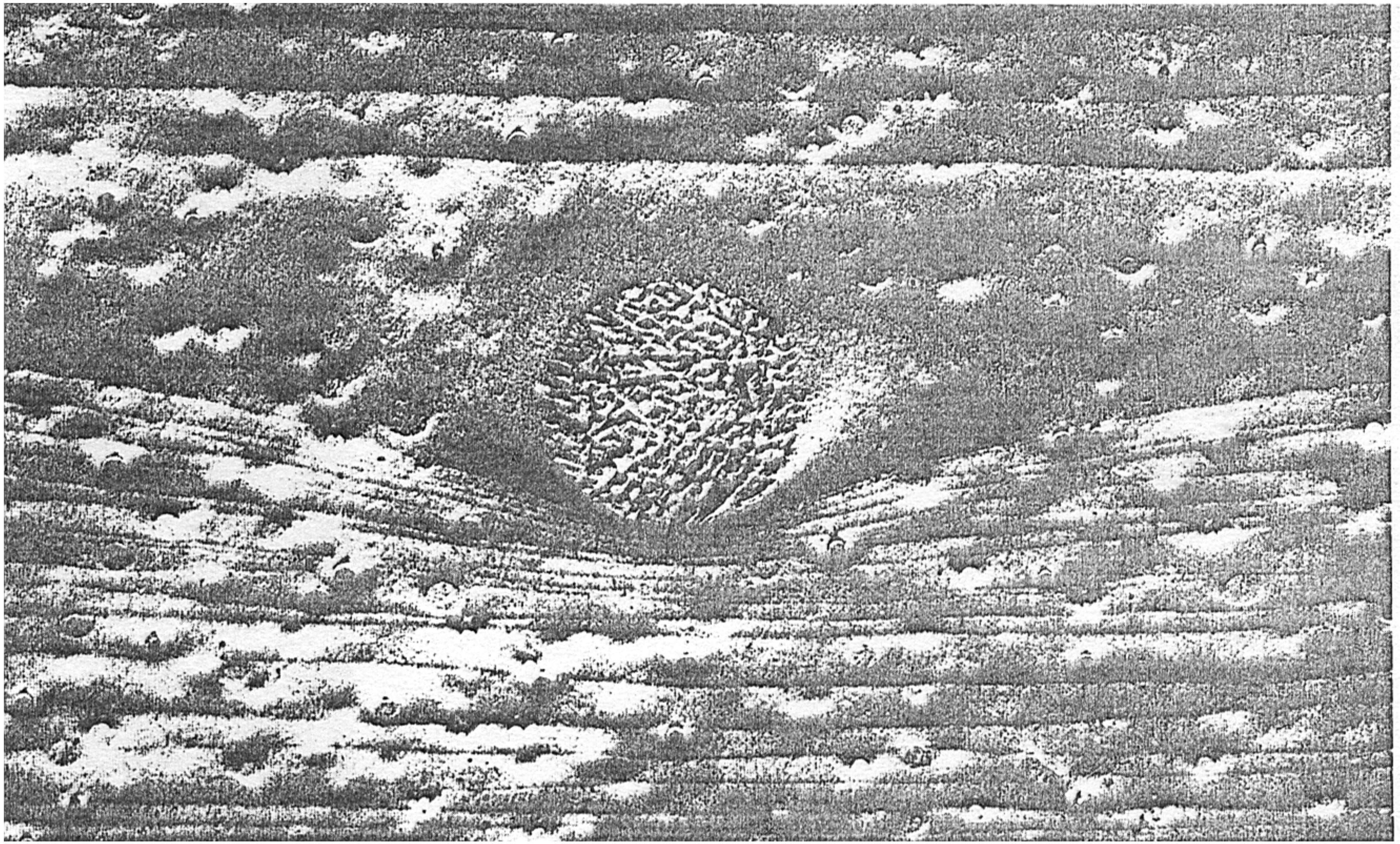
- rupture front accelerates quickly after nucleation
- terminal rupture velocity is close shear wave (mode I and III) or Rayleigh wave velocity

⟶ typical assumption: $v_r \approx h\, v_s$ with $h = 0.5 - 0.9$

rupture velocity accelerates at low stress region (capillary, low strength region)



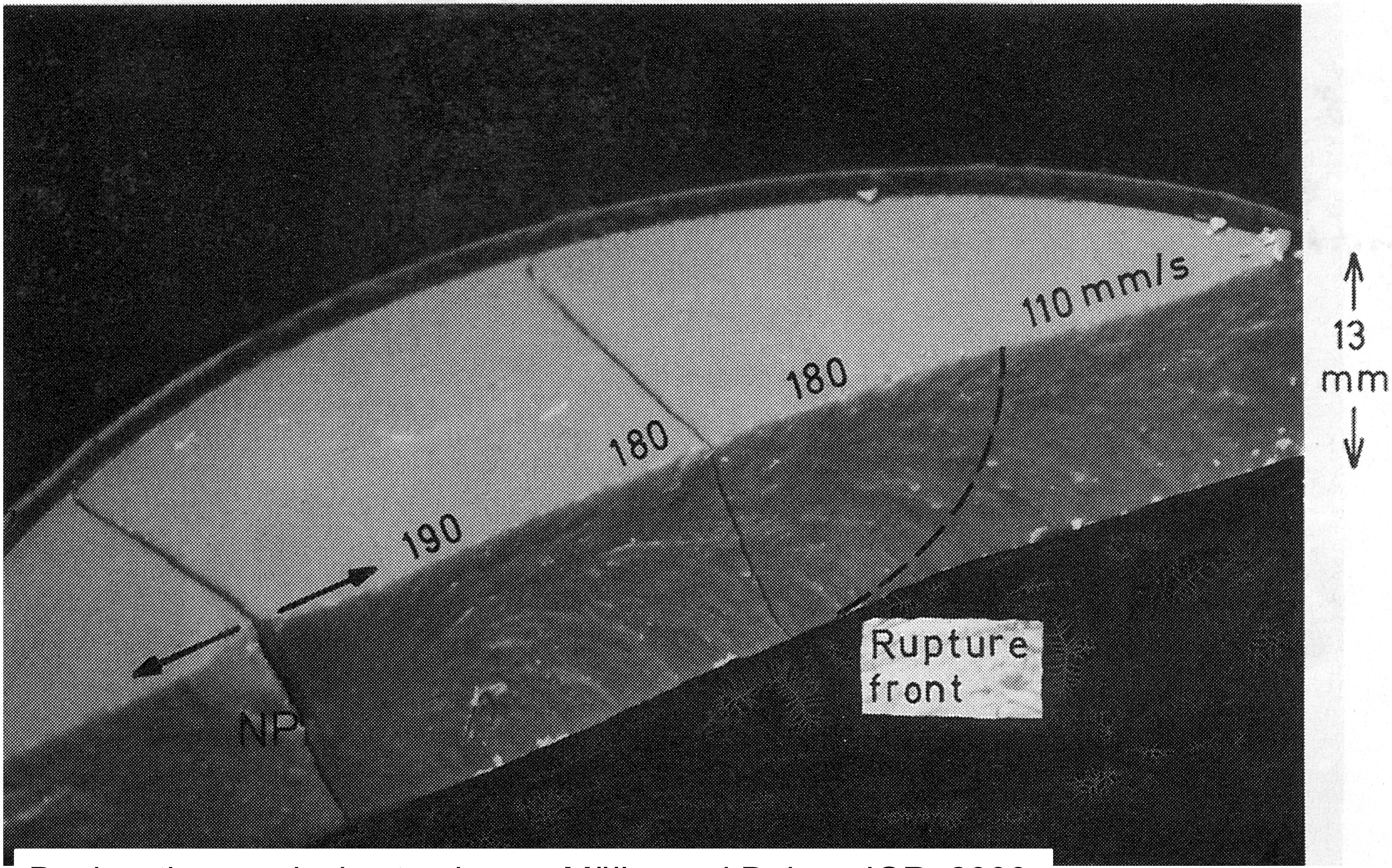
rupture velocity slows down at rigid inclusion (e.g. crystalline CaF_2 , high strength region)



How to parameterize rupture front, healing front, and variable slip ?

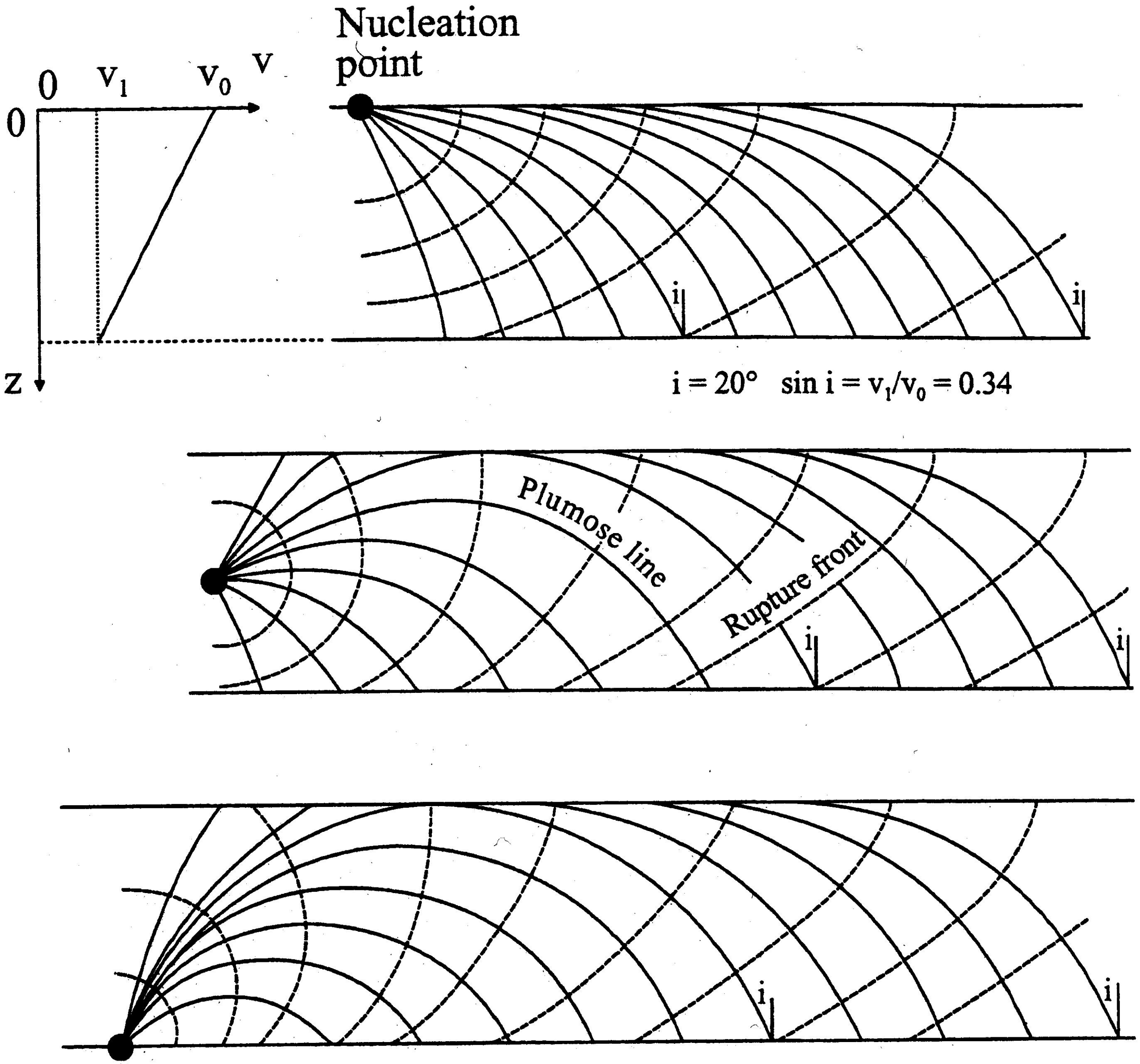
„ the Eikonal source model“

Lab experiments of tensile cracks:
plumose lines are found orthogonal to rupture fronts



Desiccation cracks in starch: see Müller and Dahm, JGR, 2000

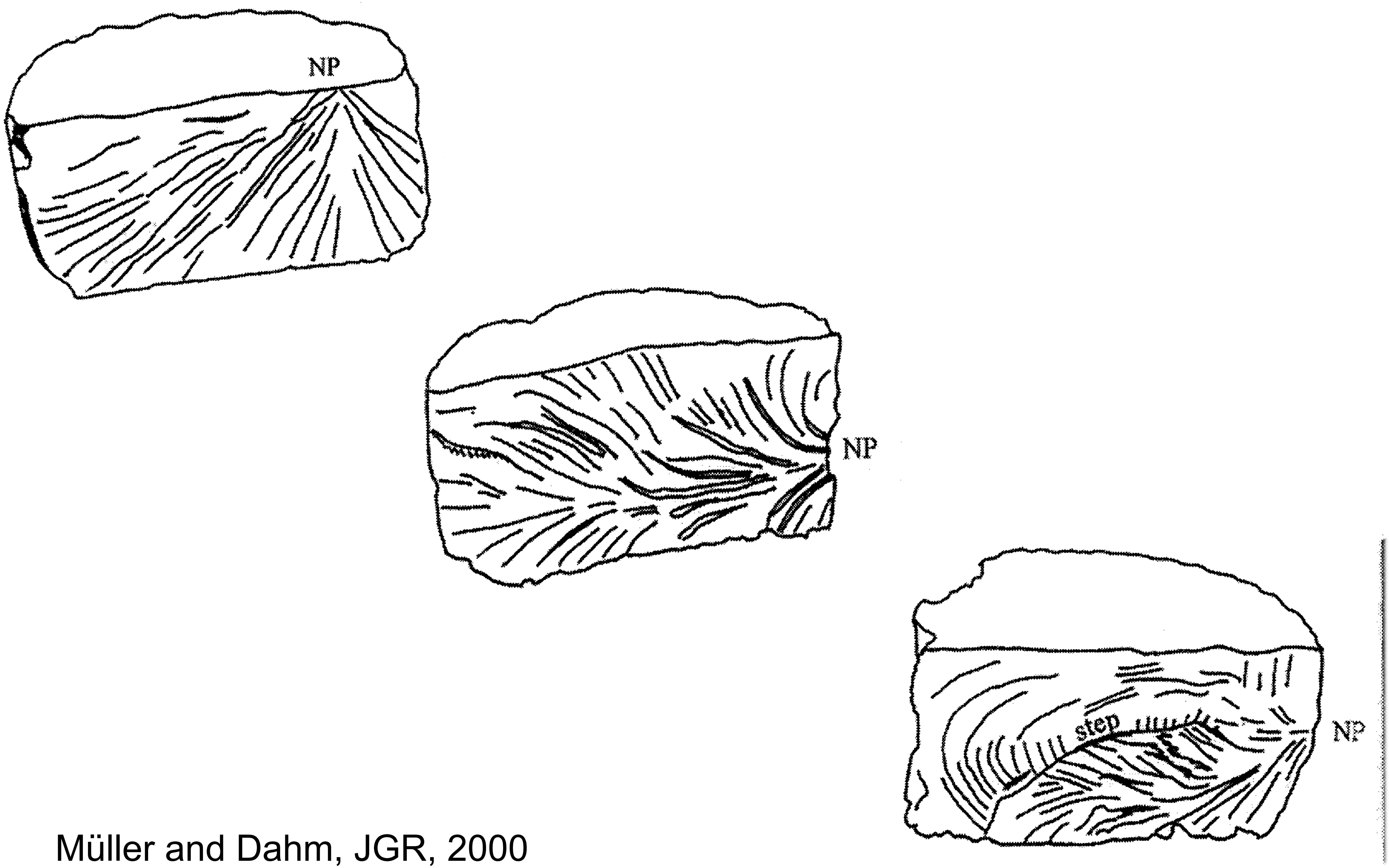
First order approach: ray - rupture front analogy



$v_r = v_0 + b\,z$

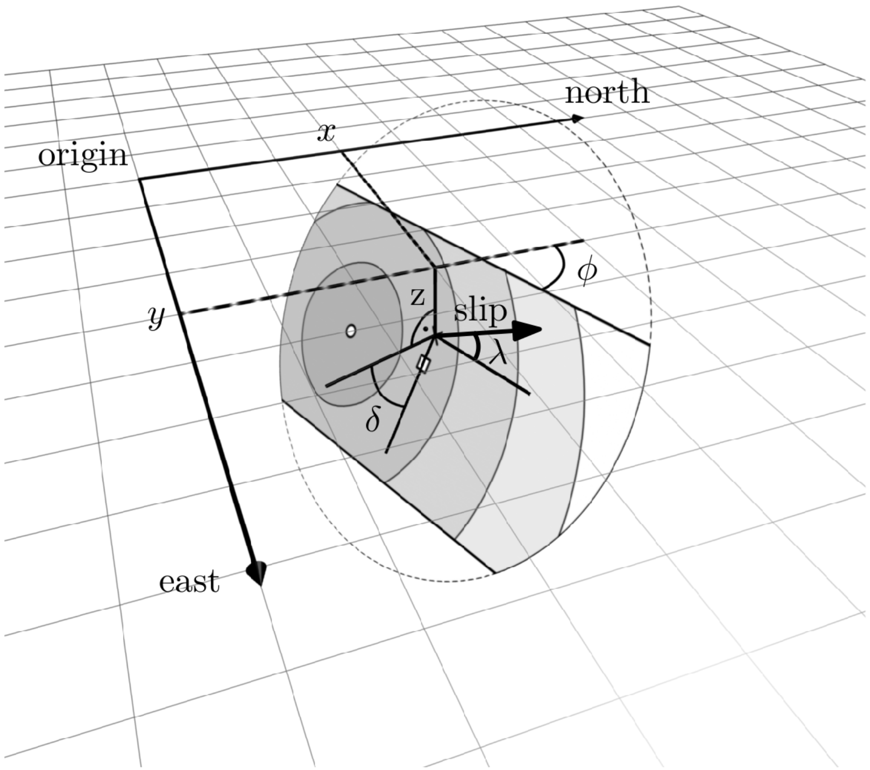
Müller and Dahm (2000)

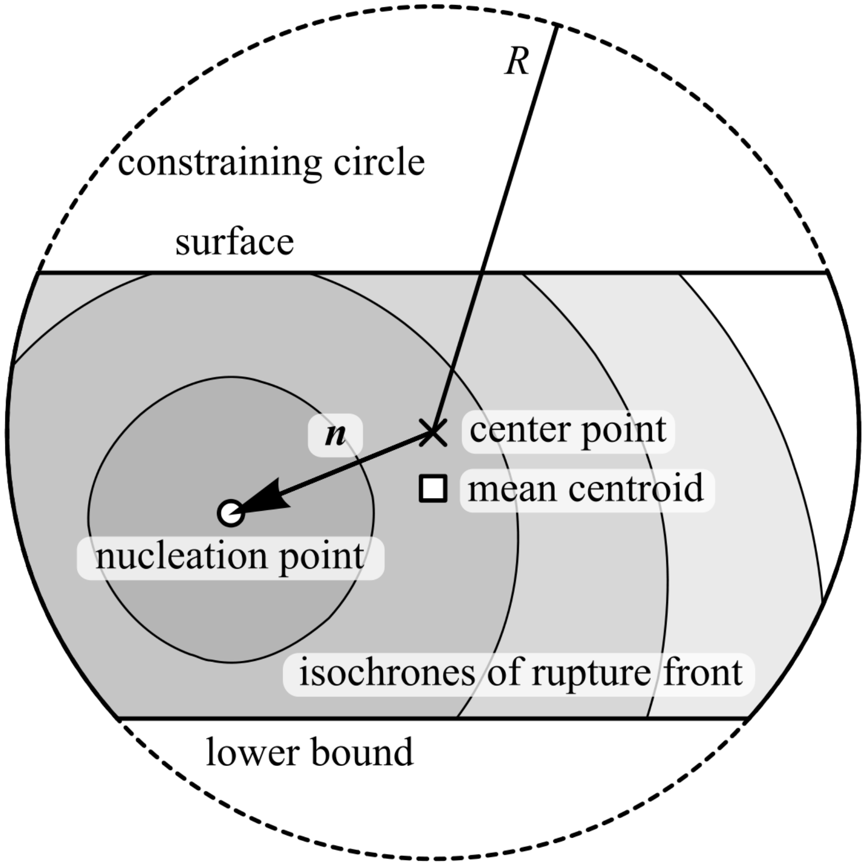
1. steps and jumps cannot be represented by ray analogy
2. reflection of rupture fronts should not be considered



Müller and Dahm, JGR, 2000

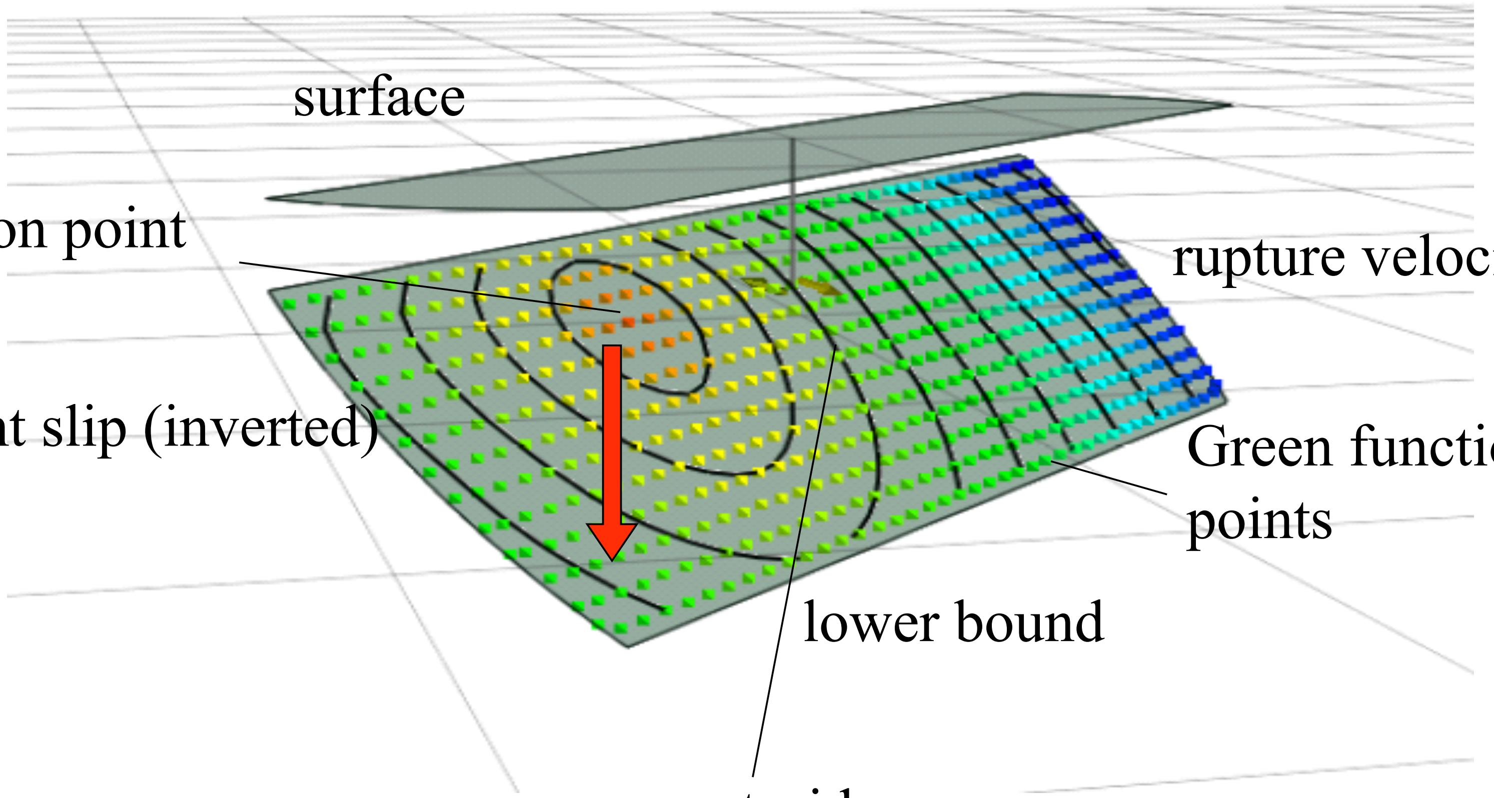
The “Eikonal source” : mathematical model





The “Eikonal source”: numerical realisation

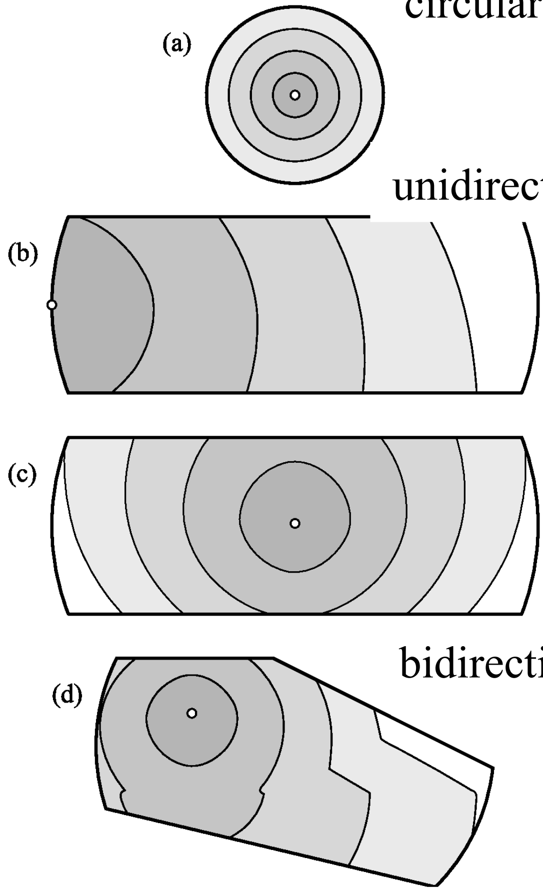
Isochorones of rupture front and of healing front from FD Eikonal solver



orientation of rupture plane (inverted)

Flexibility

- few parameter
- may consider background structural features
- may consider background wave velocity (and stress)
- flexibel
    - geometrical bounds
    - variable rupture and healing front velocity
    - variable nucelation point (asymmetric rupture)
    - (may consider variable slip)



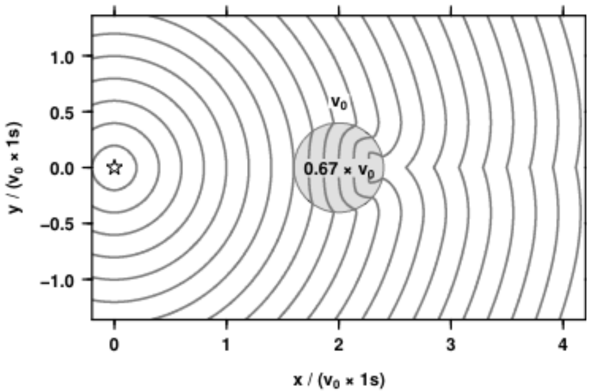
circular

unidirectional

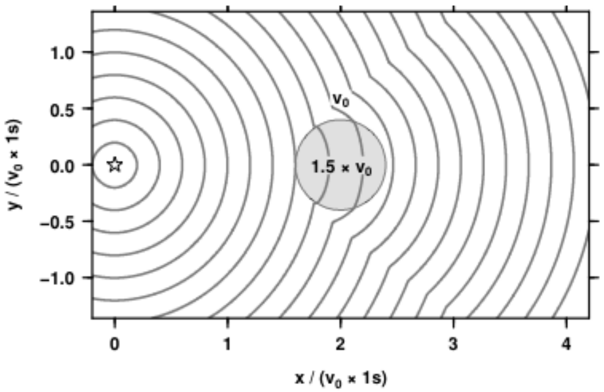
bidirectional

complex

simulation of slow patch



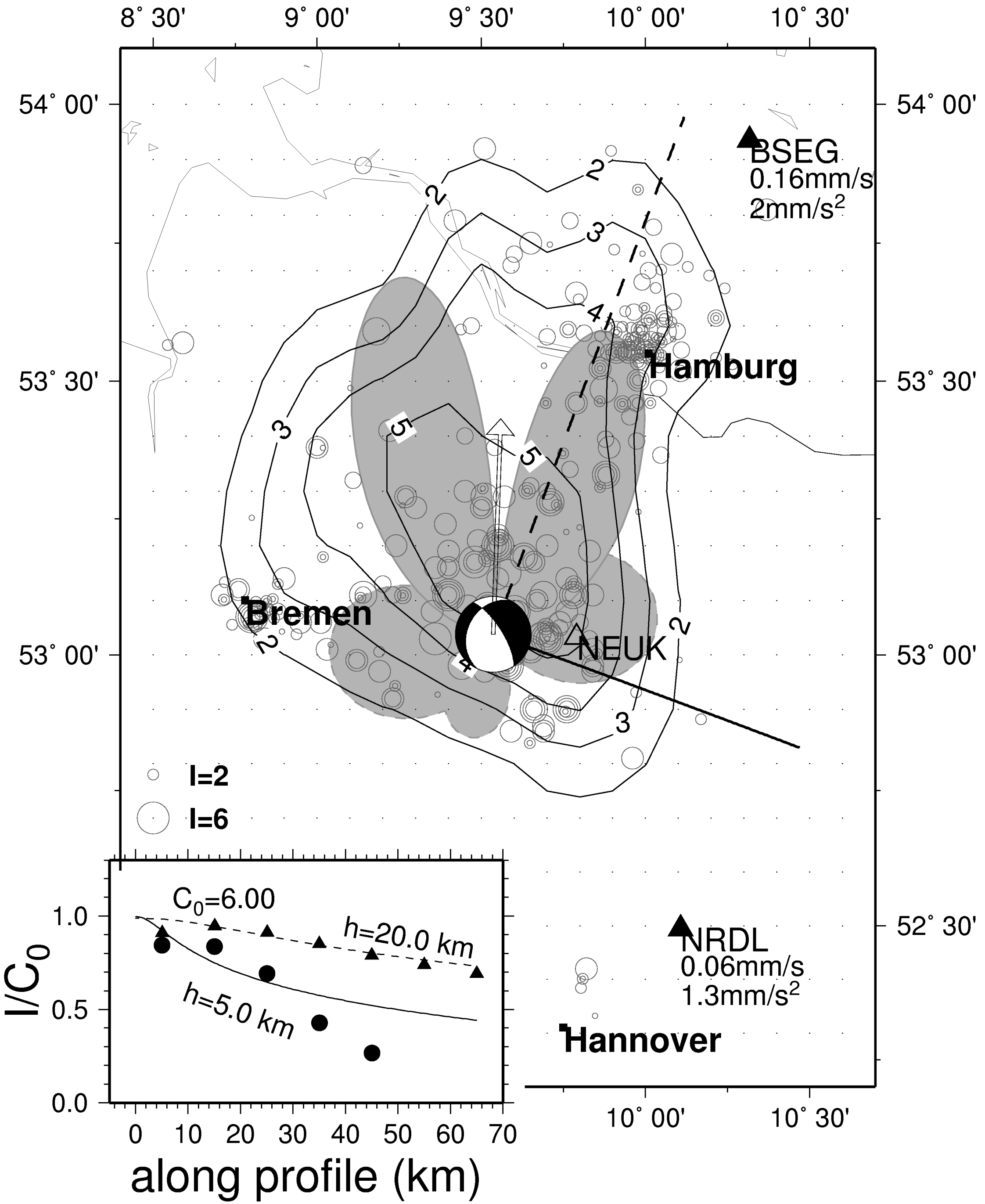
simulation of fast patch



Memo plate

- Slip may vary along rupture plane (but often assumed constant)
- Rupture velocity typically scales with shear wave velocity (but may also vary)
- Friction is important for shear cracks and can explain slip pulse ruptures
- The Eikonal source model is an empirical approach (approximation)
- Since space-time dependency of slip is constrained the Eikonal model reduces the non-uniquenes of the extended source inverse problem

Directivity effects and kinematic inversion

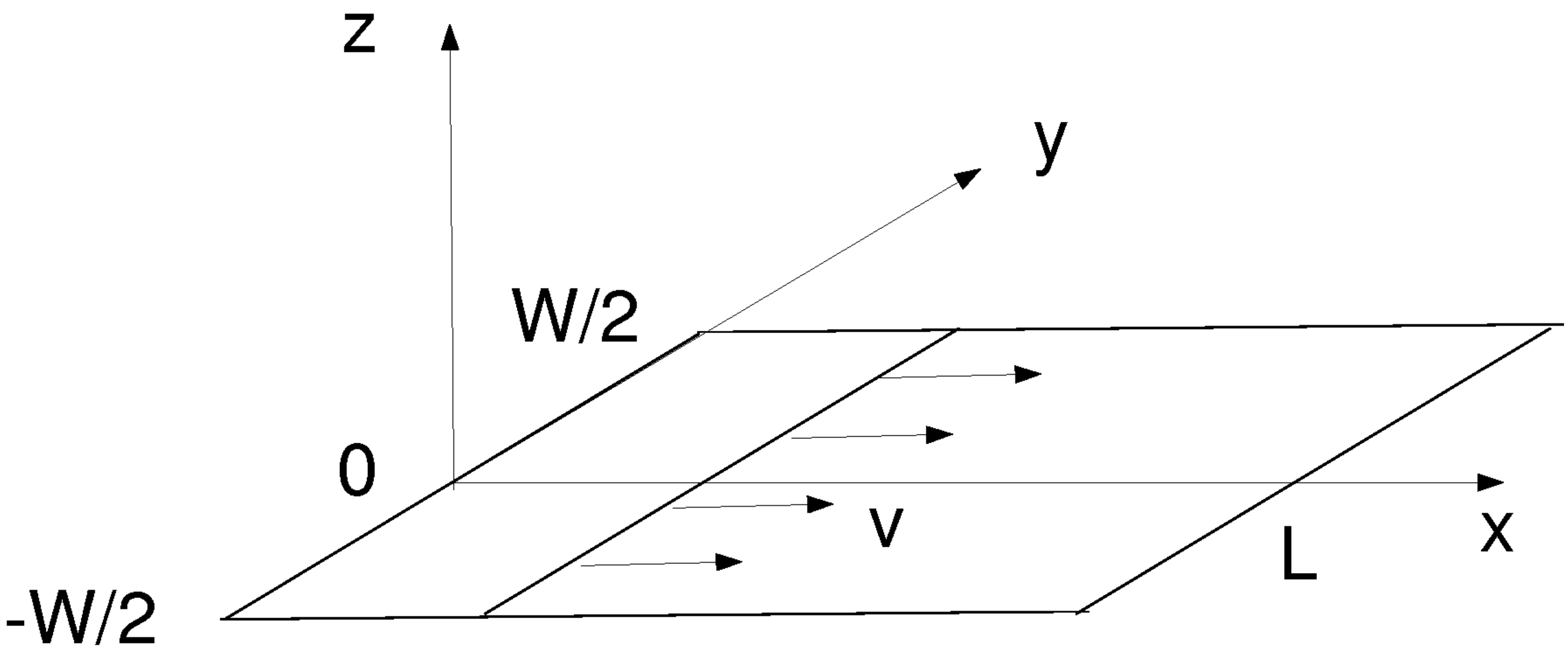


The directivity of surface waves (grey) explain why macroseismic intensity (contourlines) was higher towards north.

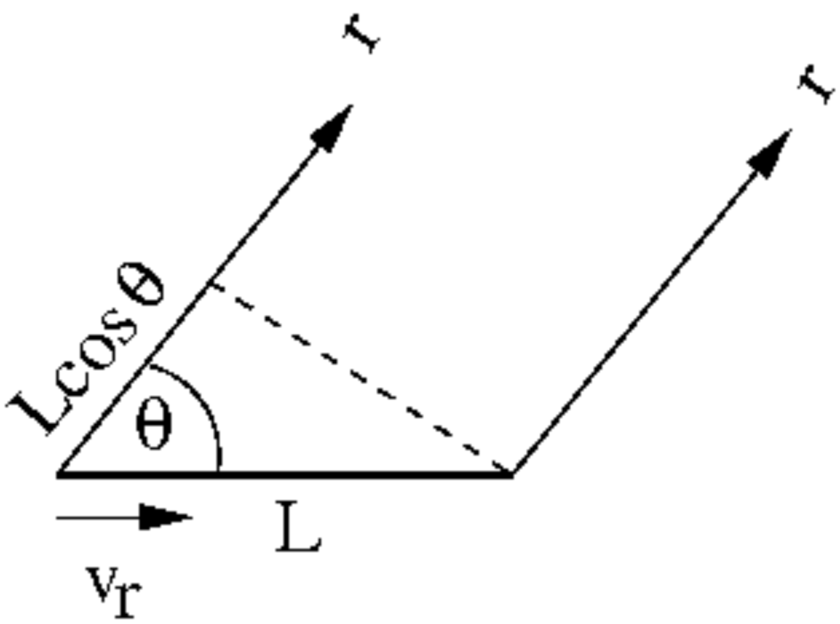
Important!!

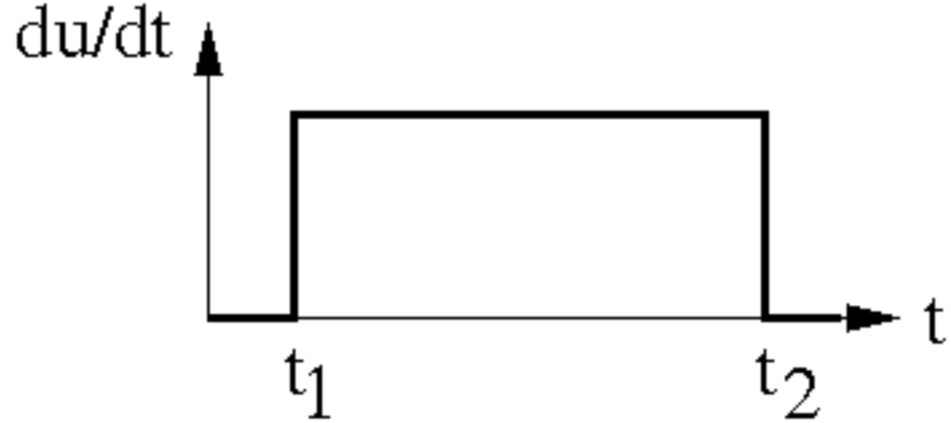
Any depth estimate from macroseismic intensities should consider the directivity effects of radiated waves.

Haskell source



unilateral propagation of line source

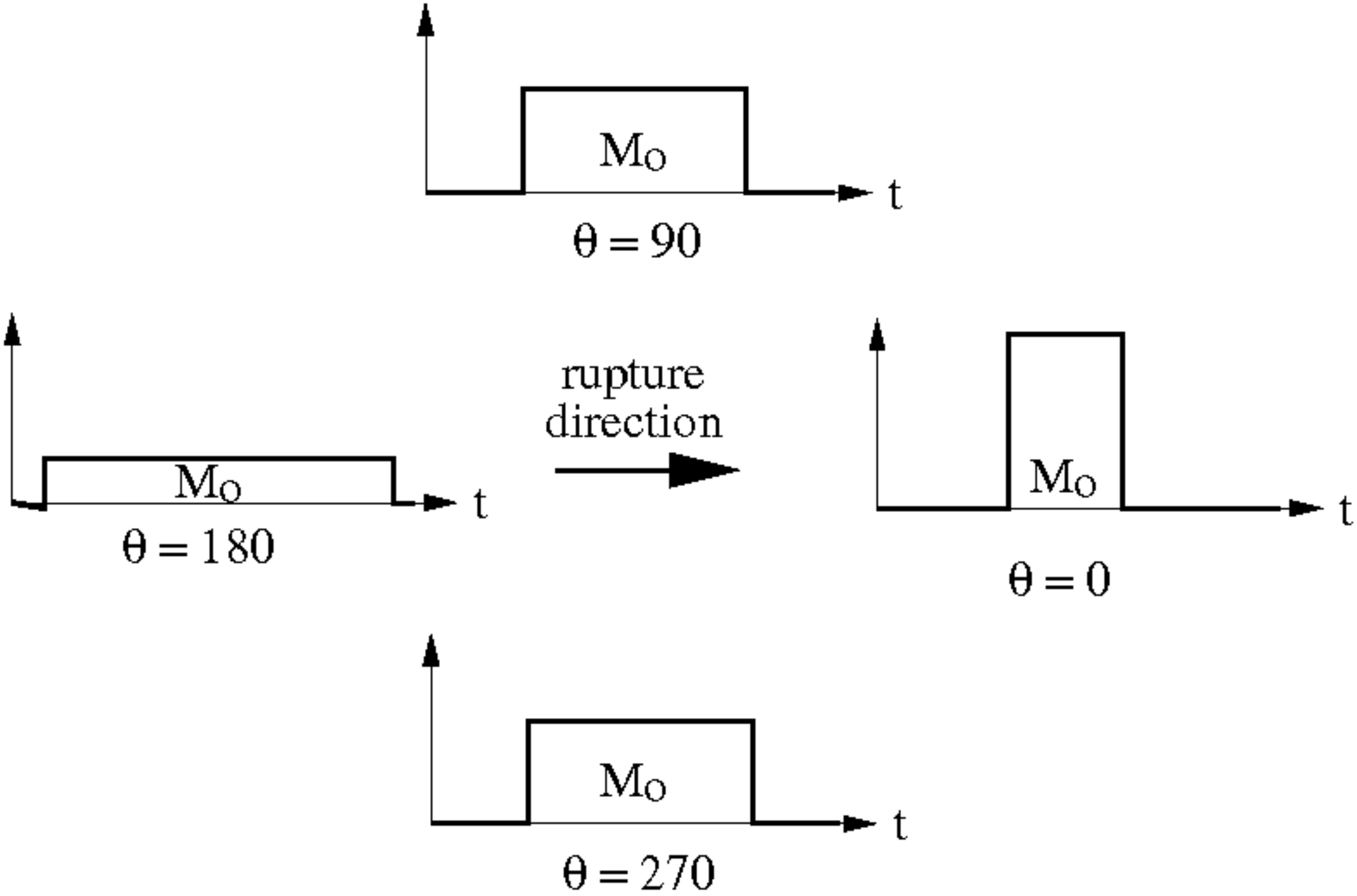




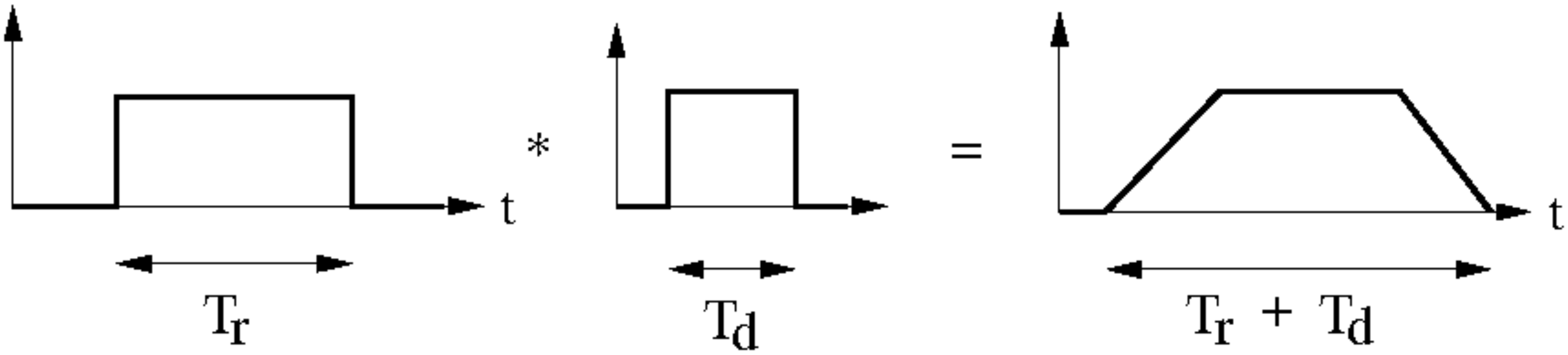
rupture duration

$$
\begin{aligned}
T_r &= t_2 - t_1 \\
&= \frac{L}{v_r} + (\frac{r}{\beta} - \frac{L\cos\Theta}{\beta}) \\
&= L(\frac{1}{v_r} - \frac{\cos\Theta}{\beta})
\end{aligned}
$$

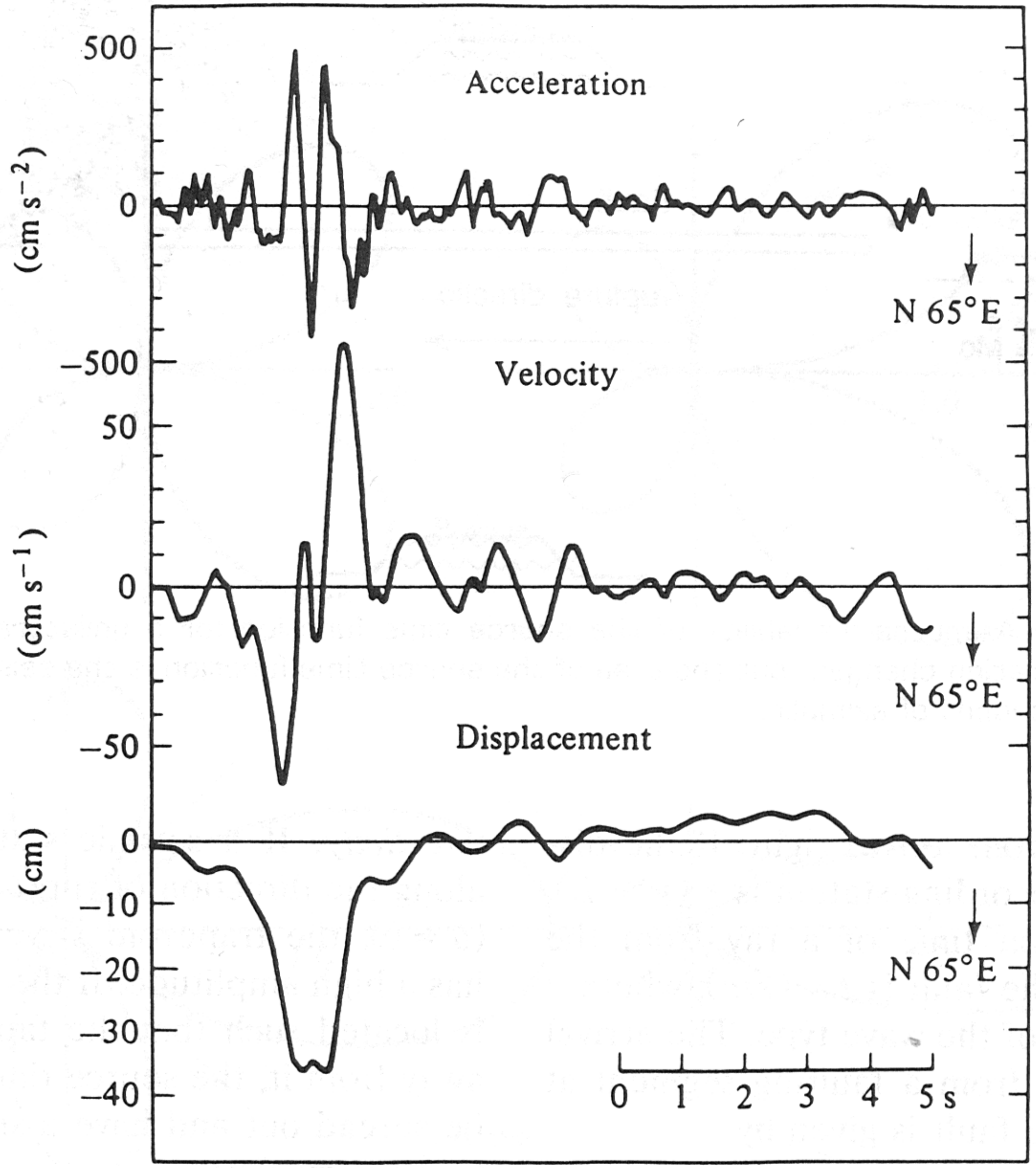
directivity of far-field pulse



Rupture duration and slip duration
= trapezoidal displacement pulses



trapezoidal slip



SH Ground motion near the Epicenter of an earthquake at Parkfield. SH radiaiton is maximal, P-waves are nodal (Aki, 1968)

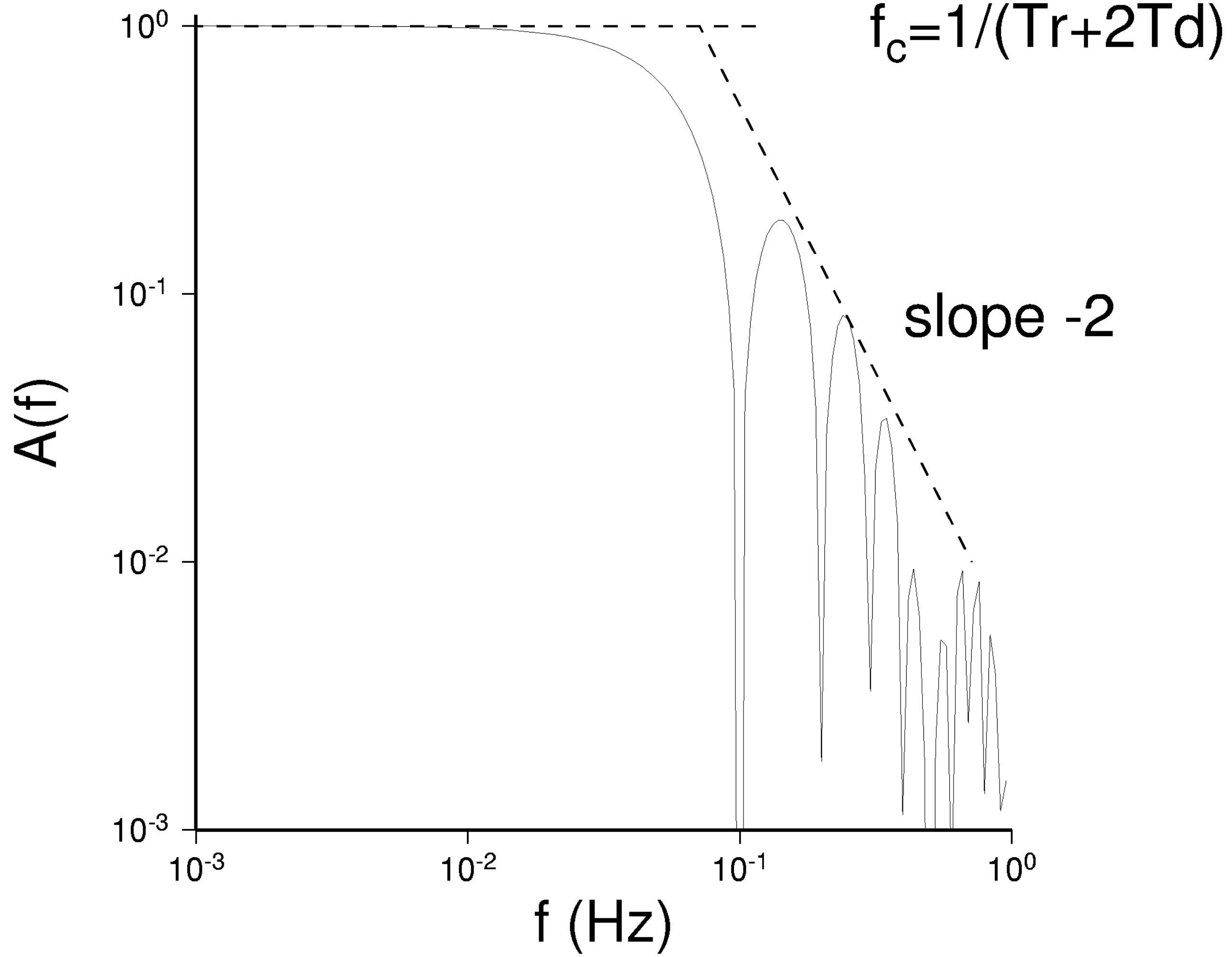
source spectra

The convolution of two boxcar function leads
In frequency domain to a multiplication of two
sinc-functions:

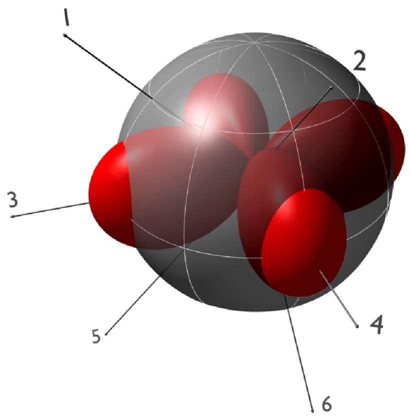
$$A(f) \;\sim\; M_0 \left|\frac{\sin \pi f T_r}{\pi f T_r}\right| \left|\frac{\sin \pi f T_d}{\pi f T_d}\right| \;\sim\; f^{-2}$$

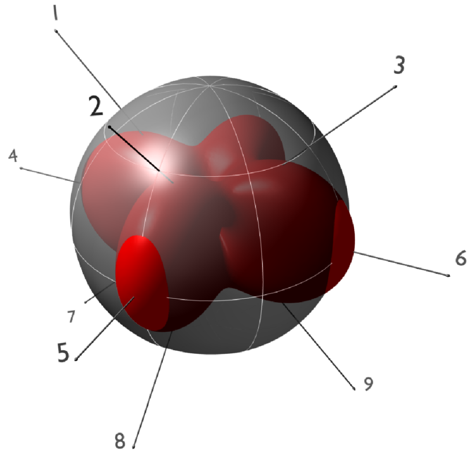
source spectra

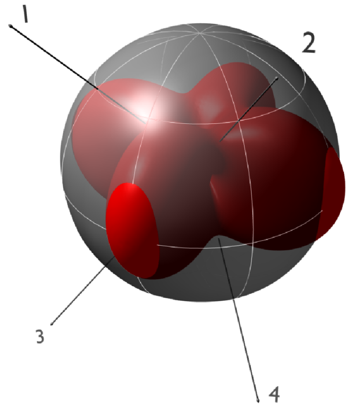
T_r=10. T_d=2.



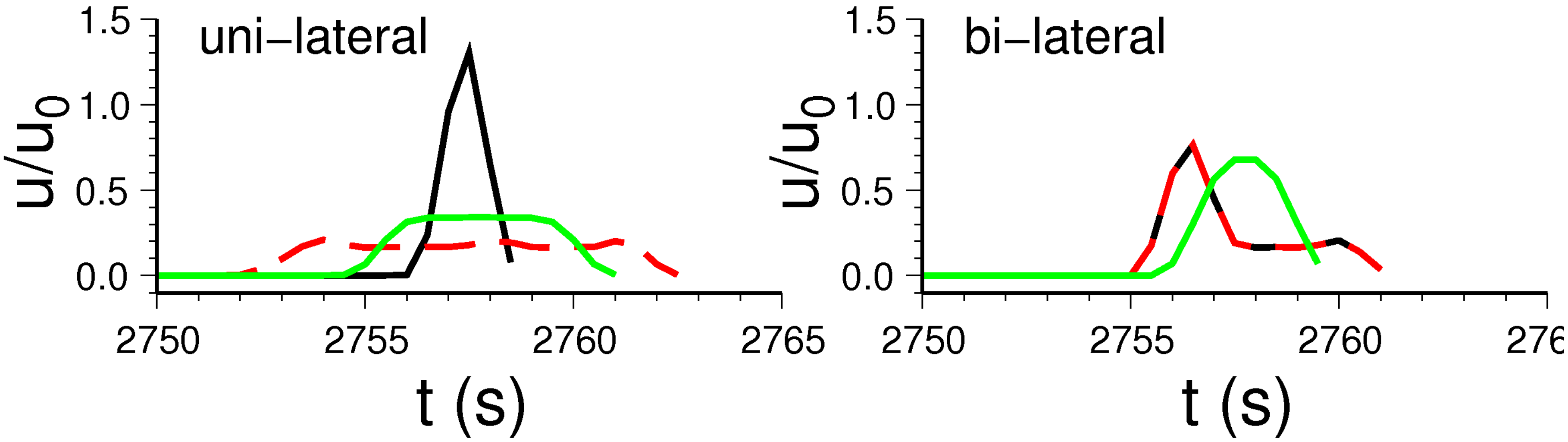
Simulations by Sebastian Heimann



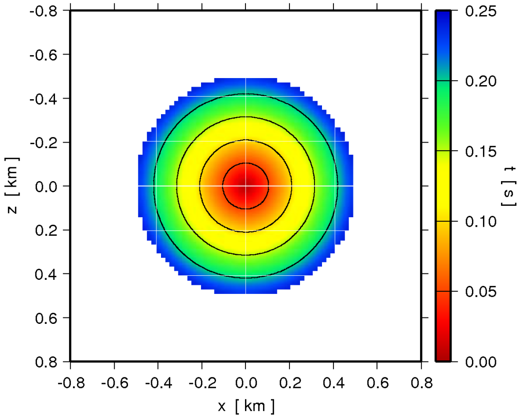


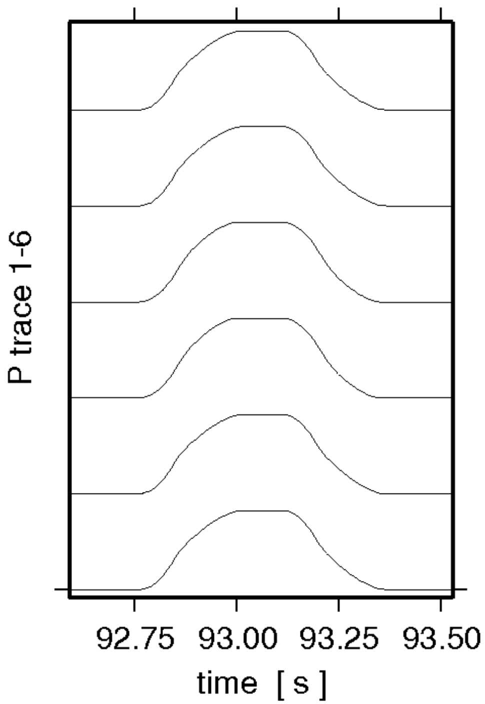


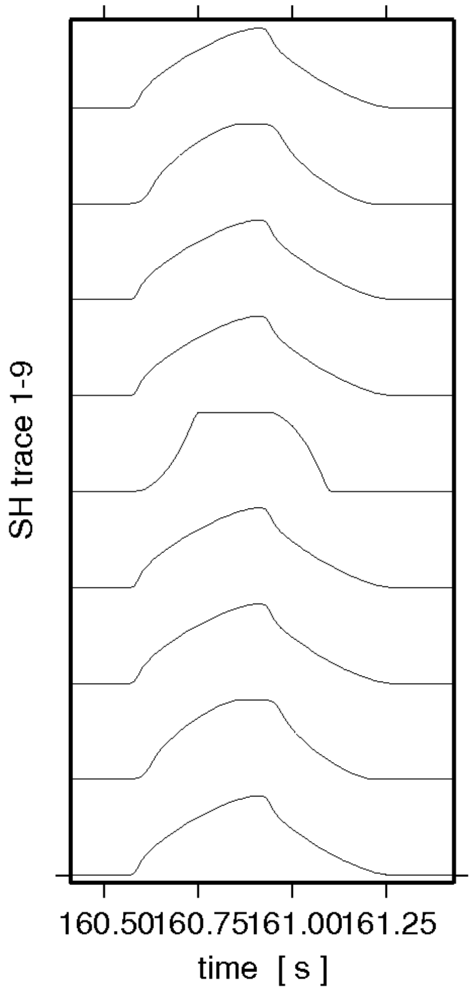
Synthetic P- and S-waves: uni- and bilateral rupture

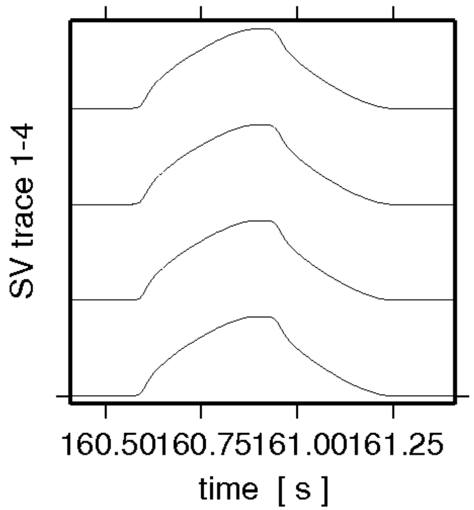


circular rupture

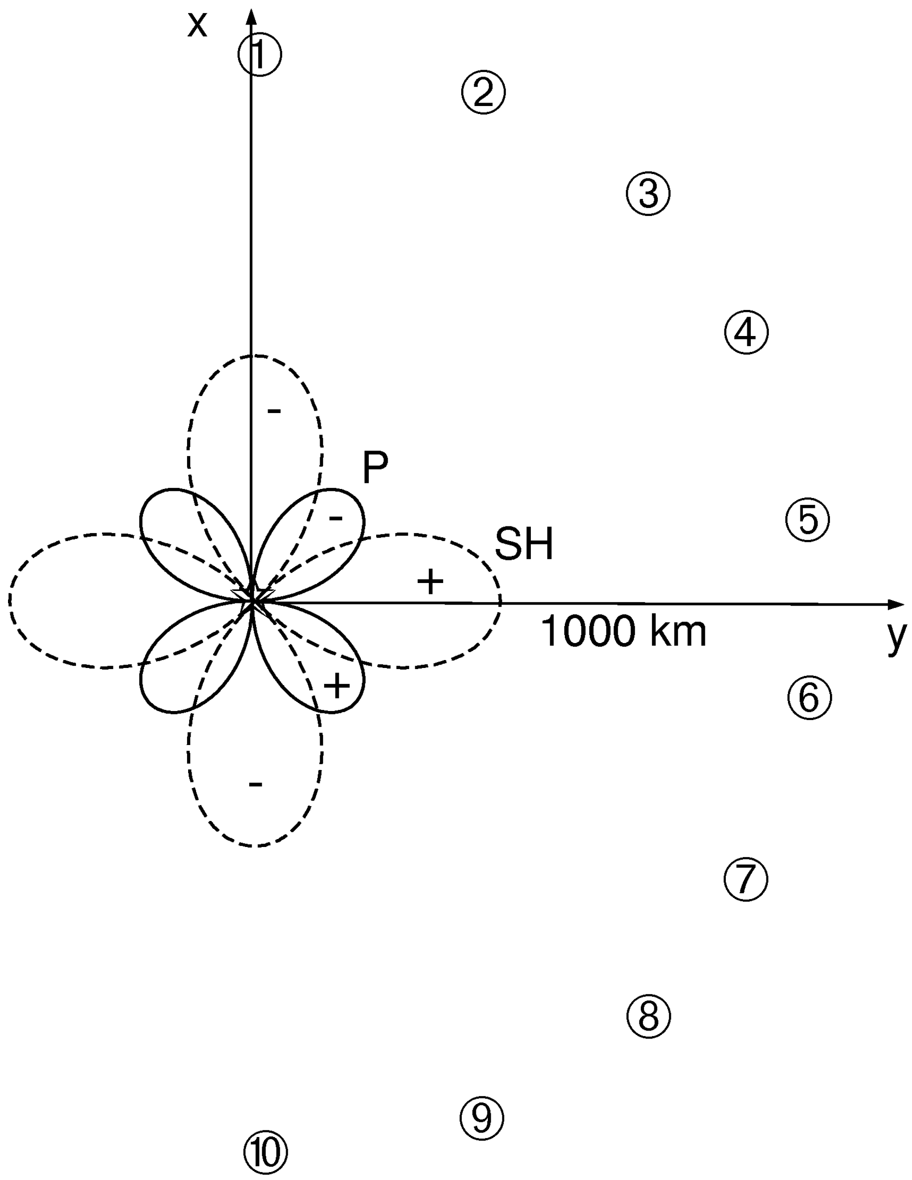




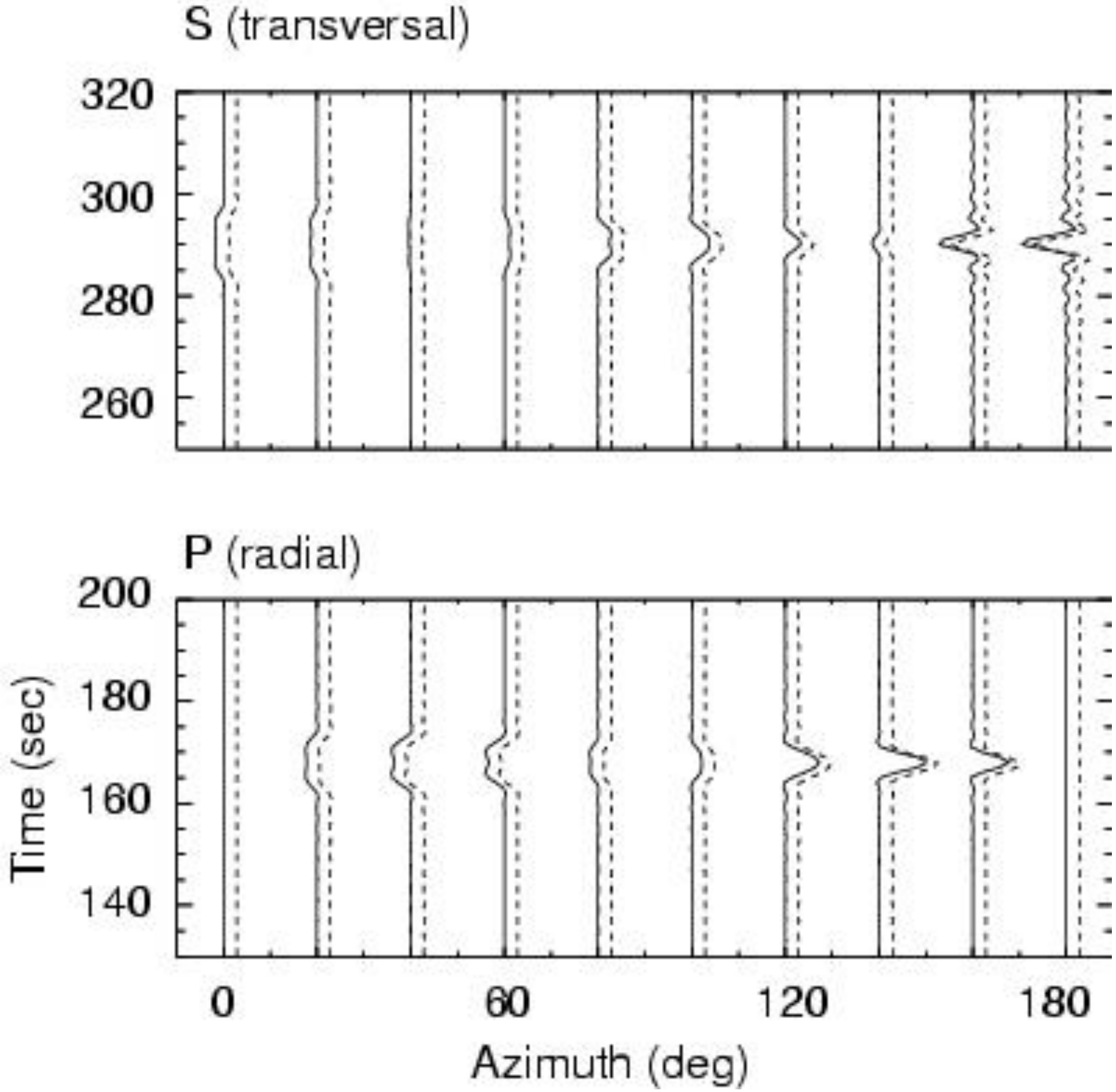




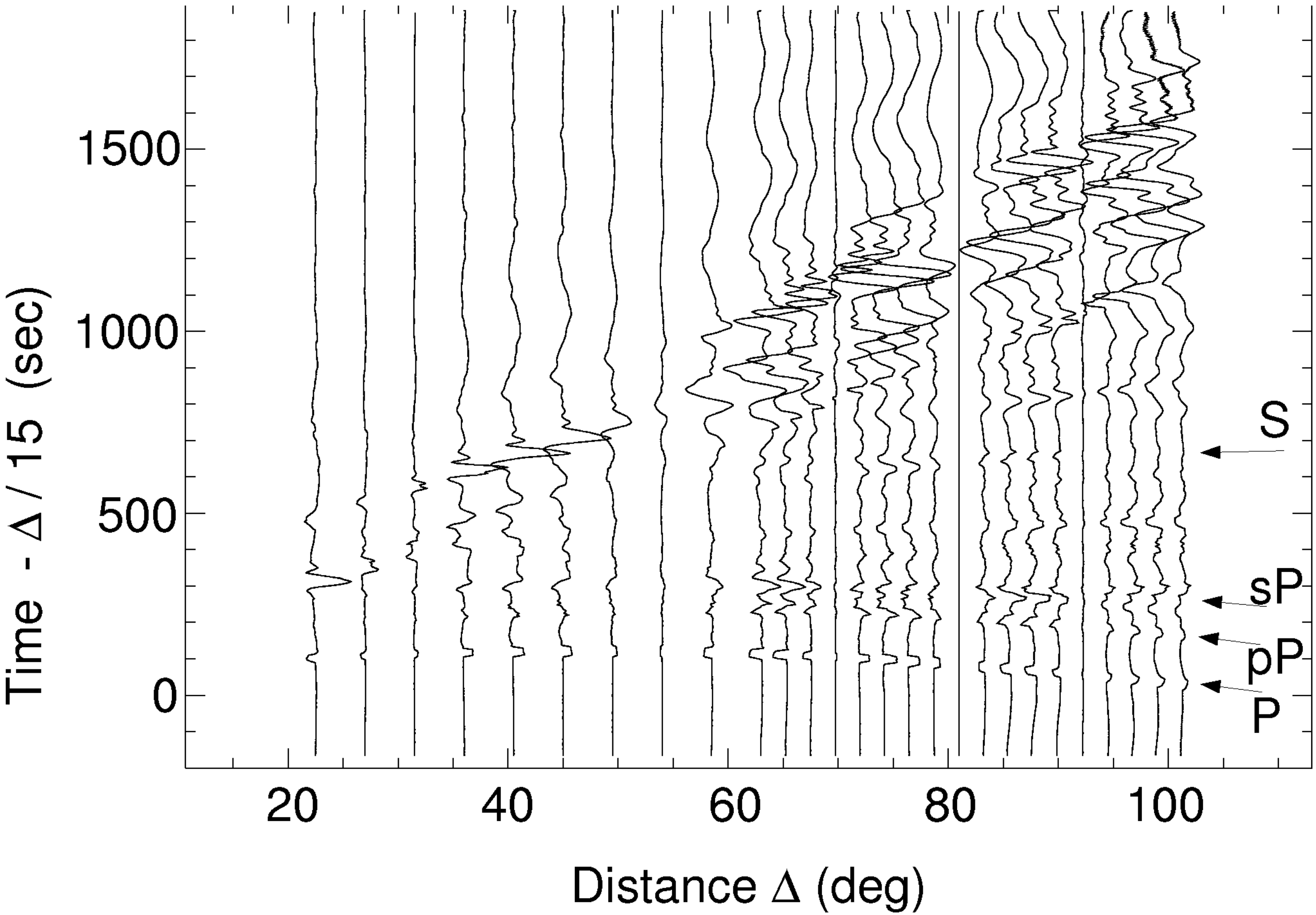
Model set up



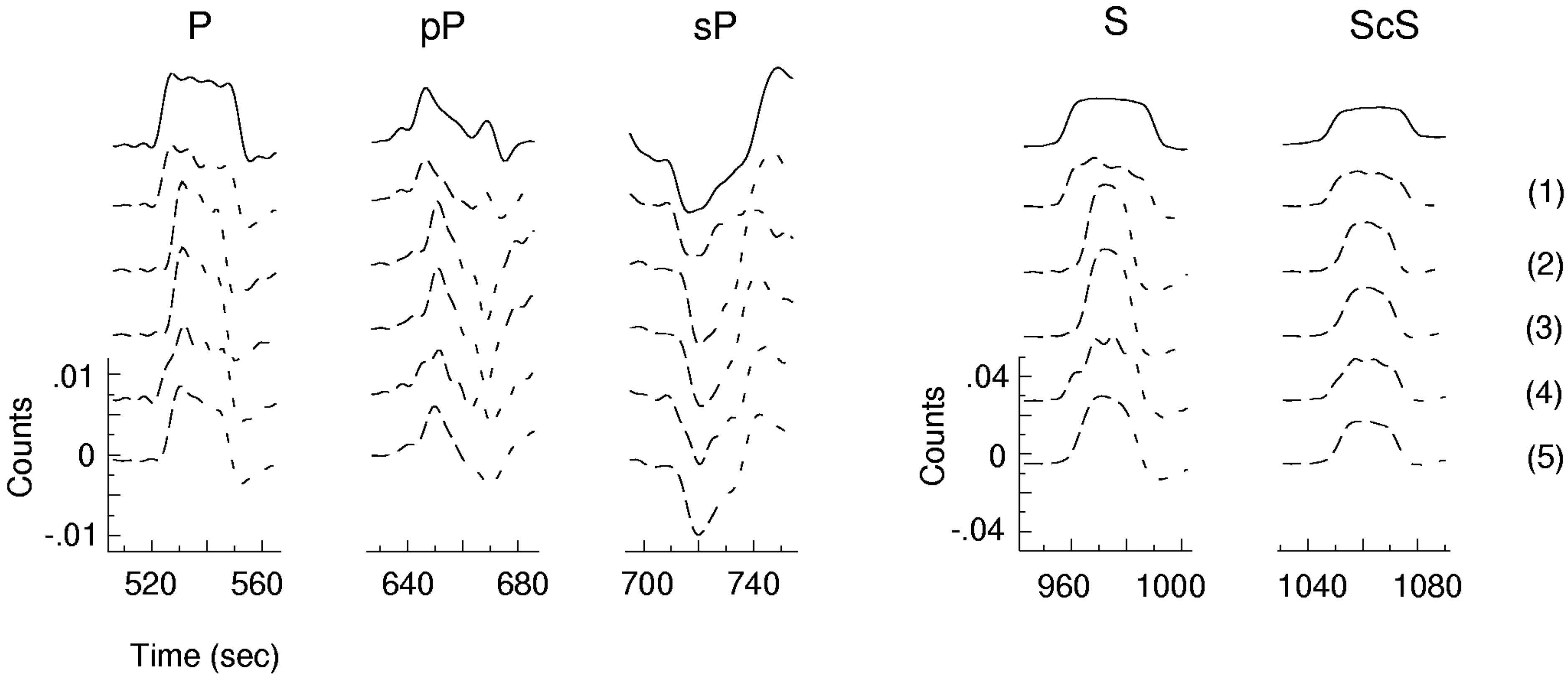
Synthetic P- and S-waves



Realistic seismograms: deep earthquake

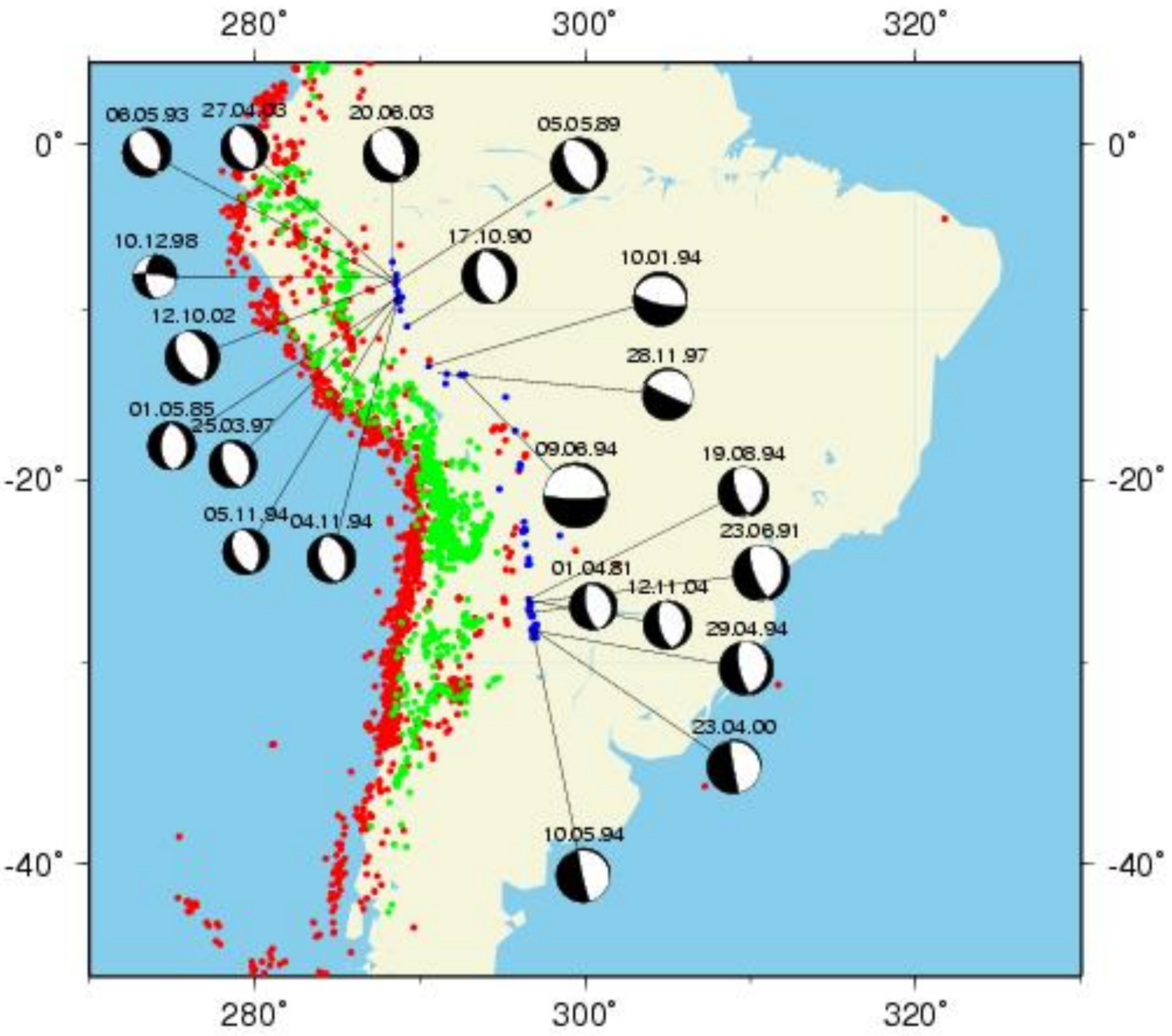


Waveforms at 58 deg epicentral distance

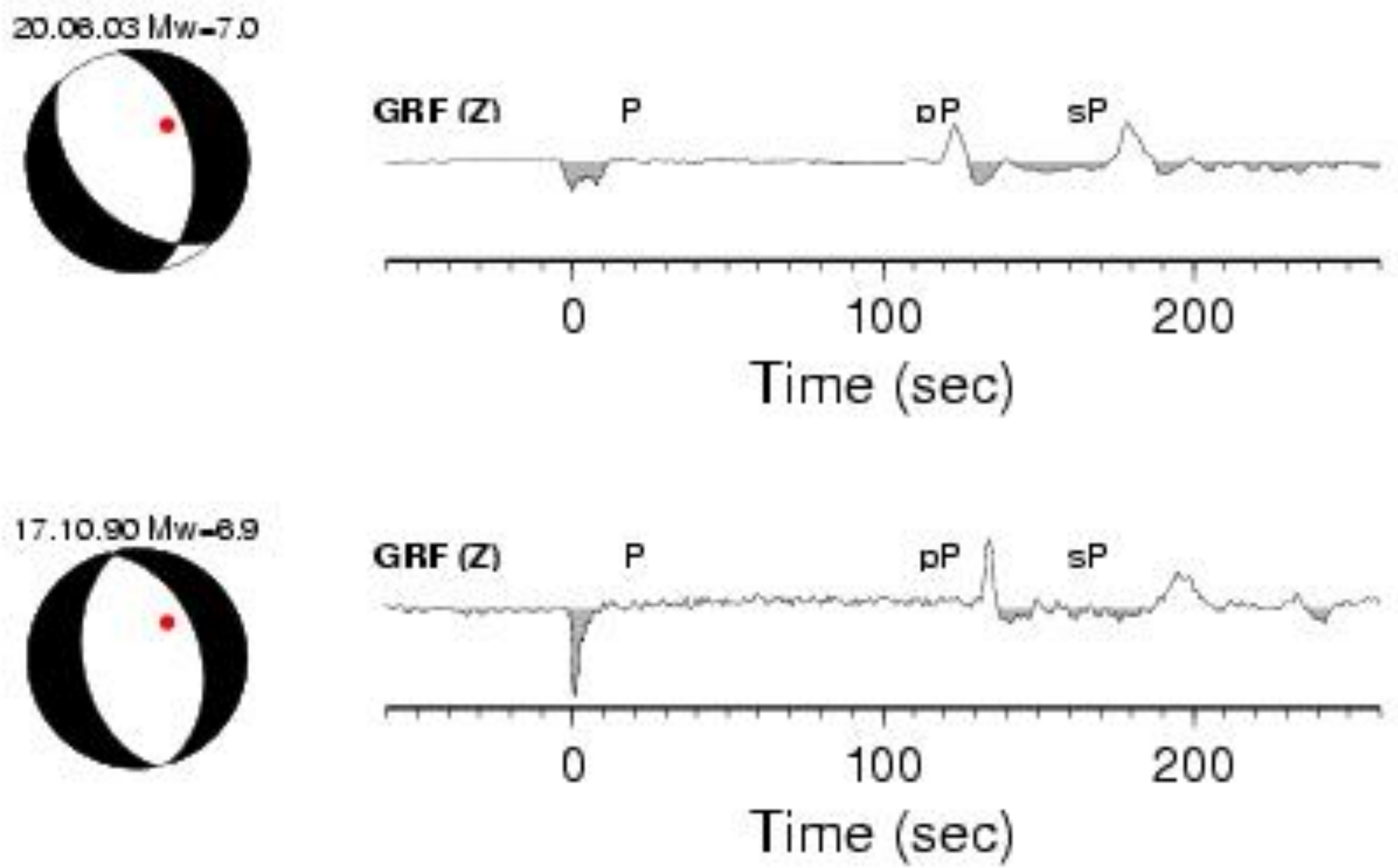


1: theoretical pulses
2: rupture in oppsoite direction
3: rupture on auxiliary plane
4: bi-drectional instead of one-directionsal
5: slip parallel to rupture instead of orthogonal

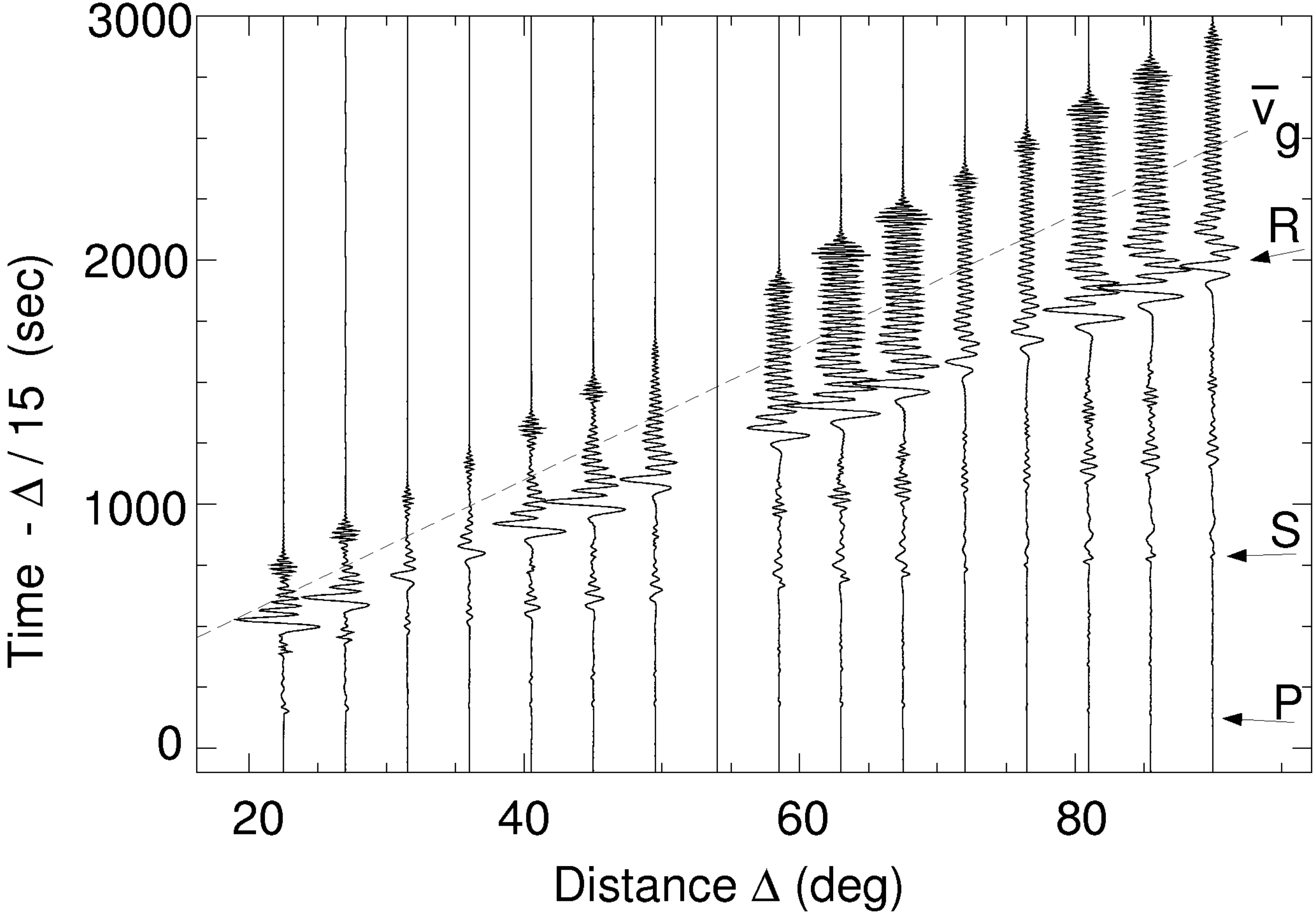
Brasilian deep focus earthquakes



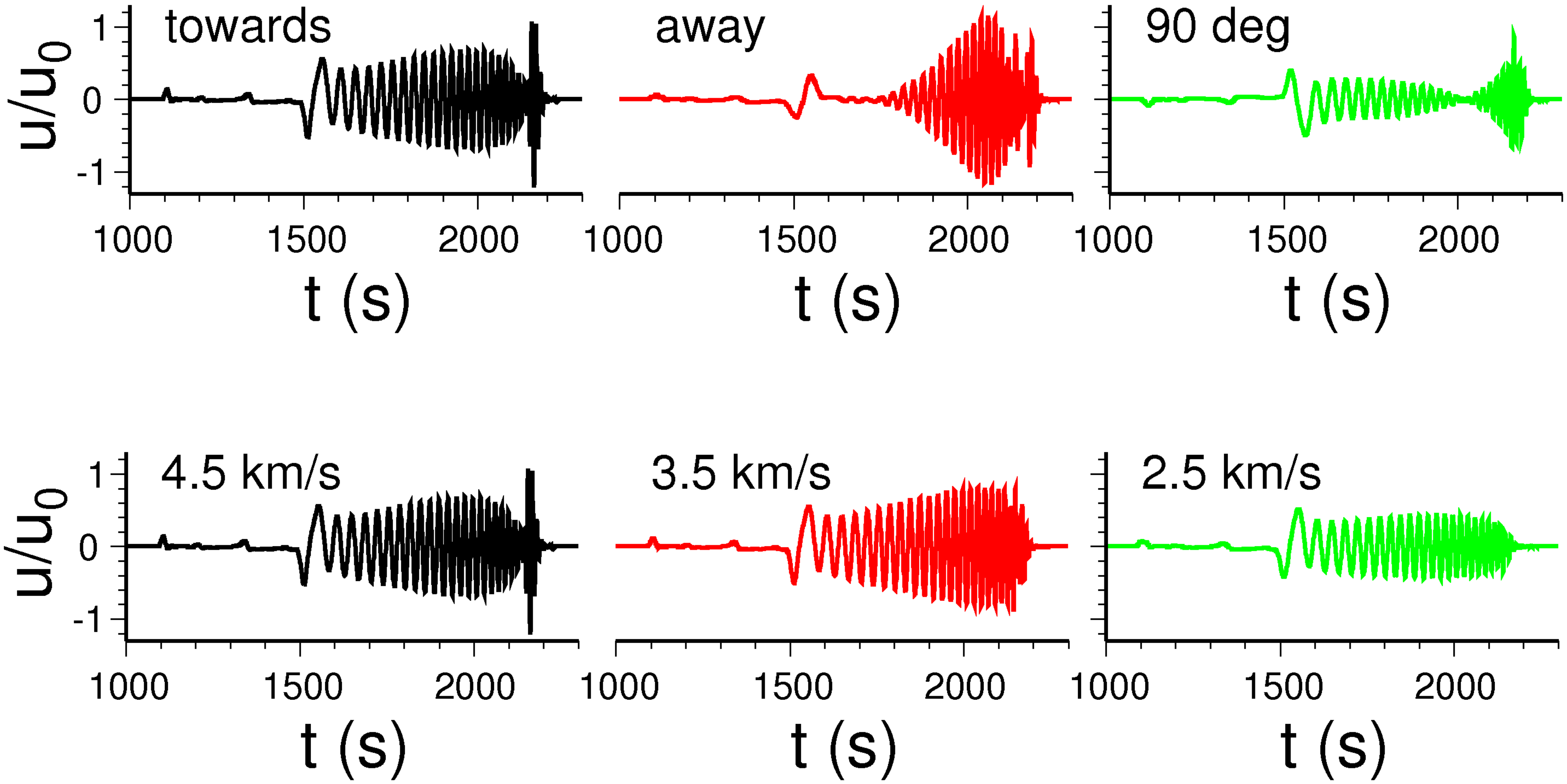
Two events with different rupture



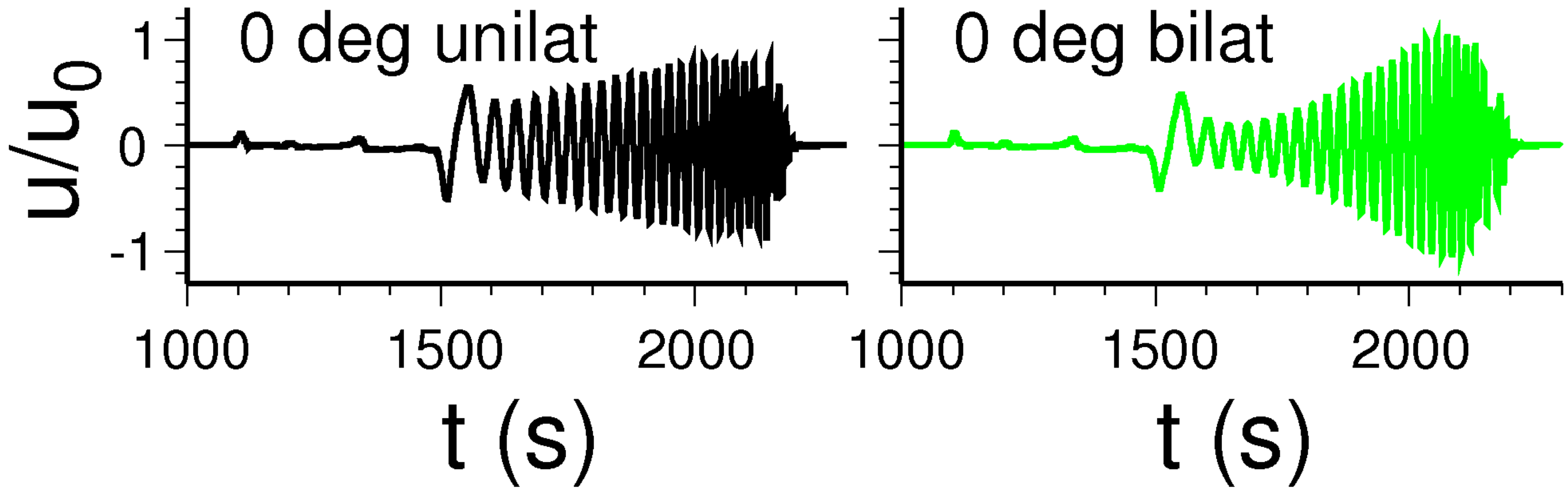
Synthetic shallow strike slip earthquake



Surface wave unilateral rupture

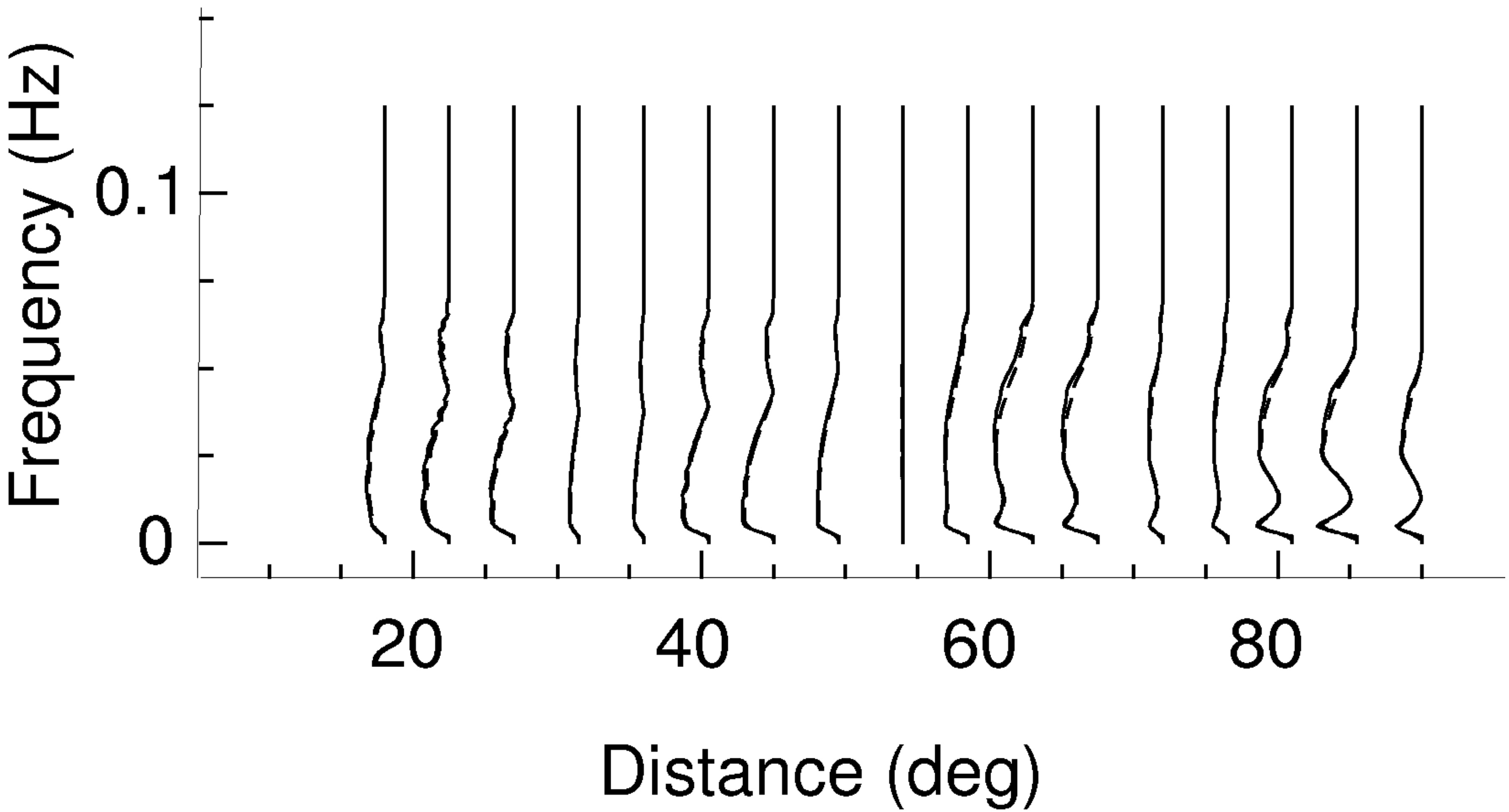


Surface wave bi- and unilateral rupture



Since total fault length (100 km) and rupture velocity (3.5km/s) was the same, the duration and the directivity pattern differed

Synthetic shallow strike slip earthquake



Test of opposite rupture direction ?